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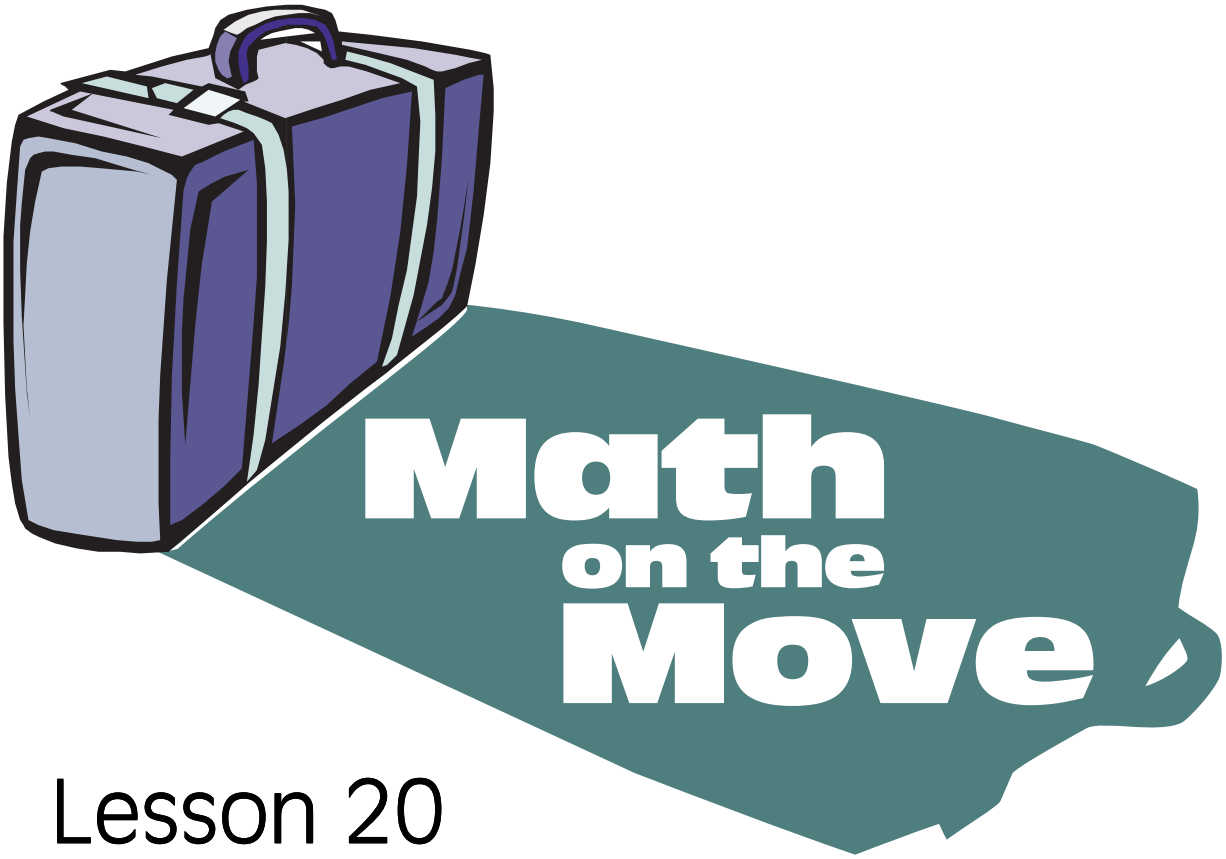

Lesson 20

Perimeter, Area, and Similarity of Triangles

Objectives

- Determine the perimeter of a triangle using algebra
- Find the area of a triangle using the formula $A = (1/2)bh$
- Find a missing side of a triangle using properties of proportion and similar figures

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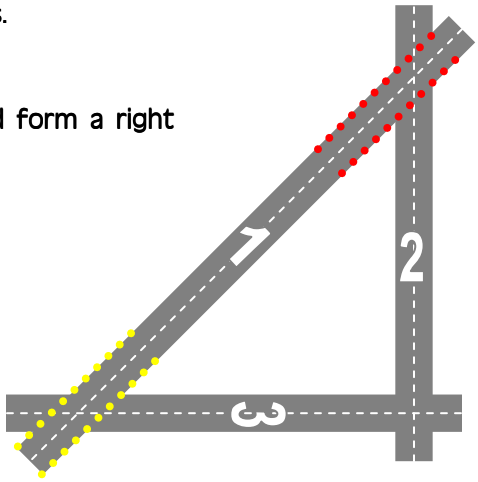
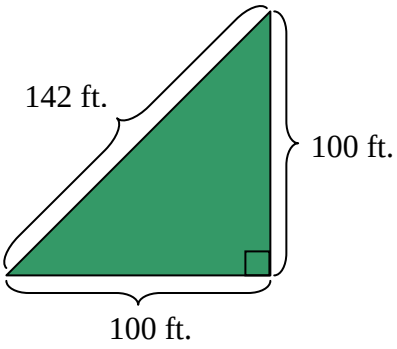
Your friend, Jorge, comes up to you all excited about his new job at the airport. You ask him what he does and he says, "I cut the grass between all the runways." You reply, "That sounds hard. That's a lot of grass to cut." Jorge says, "I get paid ten cents for each square foot I cut." You exclaim, "That's great! How many square feet are there between the runways?" He replies, "I have no idea. The runways form a triangle." Jorge then shows you a diagram of the runways. You decide to help him figure out how much money he will make cutting the grass.



Jorge explains that Runways 2 and 3 are 100 feet long and form a right angle. Runway 1 is the longest at 142 ft.

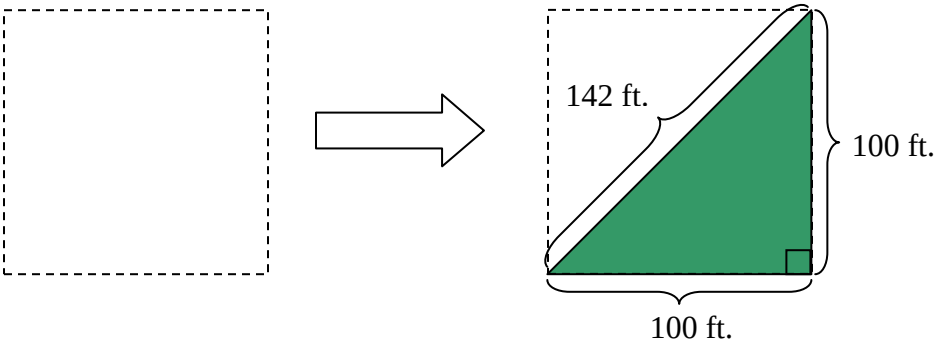
You decide to redraw this using a <u>right</u> triangle.



Think Back

A right triangle is a triangle with one right angle.

The only area formulas that you have learned so far are for squares and parallelograms. You decide to figure out the area using these familiar shapes. You notice that the two sides of equal length form a right angle. This reminds you of a square, so you take a square and line it up with the triangle.



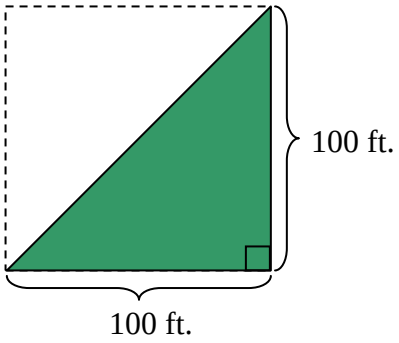
You see that the two sides of the triangle line up perfectly with the square. You also notice that the longest side looks as if it cuts the square in half. You can see that the area of the triangle is one-half the area of the square.

The area of the square is base times height.

$$\text{Area} = b \cdot h = (100 \text{ ft.})(100 \text{ ft.}) = 10{,}000 \text{ sq. ft.}$$

If the area of the triangle is one-half of that, then

$$\text{Area of triangle} = 5{,}000 \text{ sq. ft.}$$

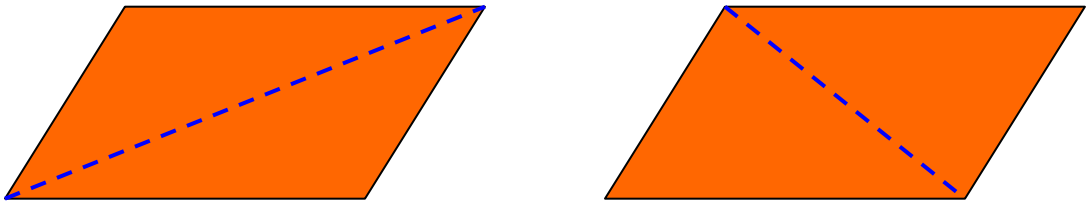


So, Jorge will make $5{,}000 \times \$.10 = \$500$ for cutting the grass.

You discovered that the given triangle was made by cutting a square in half. In fact, all triangles can be made by drawing the **diagonal** of a parallelogram.

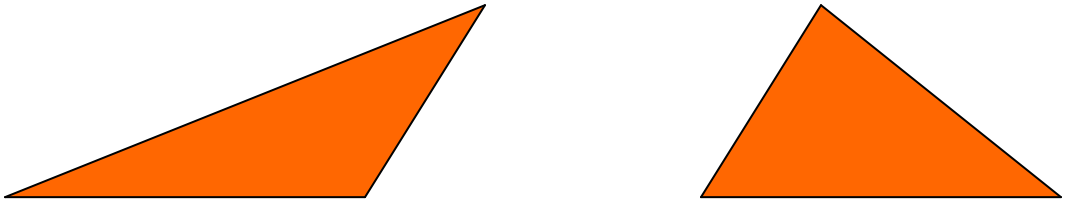
- A **diagonal** of a polygon is any line segment, other than a side, that connects two vertices.

In the following parallelogram, two diagonals can be drawn.

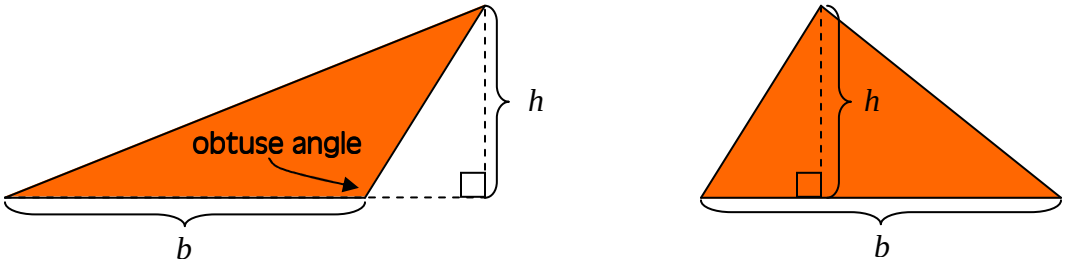


Each of these diagonals cuts the parallelogram in half, forming two triangles. Since the diagonal cuts the parallelogram in half, the area of each triangle formed is one-half of the parallelogram. If the area formula for a parallelogram is $A = b \cdot h$, then the area of a triangle is

$$Area\ of\ a\ Triangle = \frac{1}{2} \cdot b \cdot h$$



These are the two triangles formed by the parallelogram. They have the same area, because they cut the same parallelogram in half. These triangles share the same base and height.



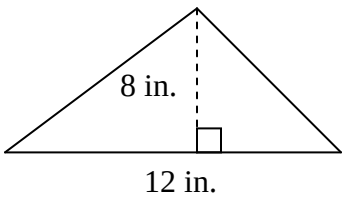
The height of the triangle is the perpendicular line segment drawn from the base to the vertex opposite the base. Notice that the height of the above left triangle is drawn outside the triangle. In order to draw the height of an obtuse triangle, you must extend the base as shown above.

Example

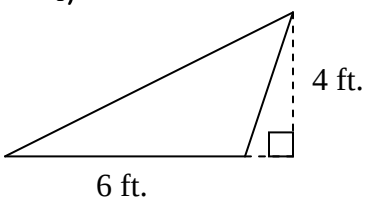
Find the area of the following triangles.

a)

b)



c)



Solution

To answer these problems, we need to remember that the area formula for a triangle is

$$A = \frac{1}{2}bh$$

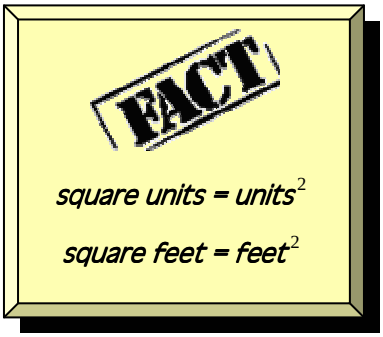
Think Back

When variables and numbers are written next to each other, it means multiplication.

a) This is a right triangle with a base of 2.5 units and a height of 1 unit. So the area is

$$A = \frac{1}{2}(2.5)(1) = 1.25 \text{ square units}.$$

Don't forget to include units in your answer.



b) In this triangle, the base is 12 in. and the height is 8 in. The area is

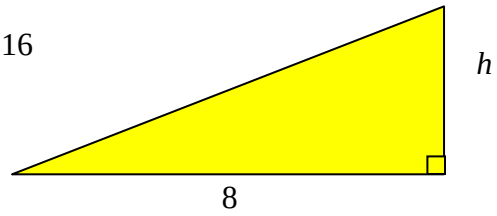
$$A = \frac{1}{2}(12)(8) = 48 \text{ square inches}.$$

c) In the last triangle, the base is 6 ft. and the height is 4 ft. The area is

$$A = \frac{1}{2}(6)(4) = 12 \text{ square feet}.$$

Example

Find the missing height of the triangle.



Solution

We are given the length of the base, as well as the area. We need to find the height. If we recall our formula for the area of a triangle,

$A = \frac{1}{2} \times b \times h$, and we substitute the dimensions we are given, we get,

$$16 = \frac{1}{2} \times 8 \times h$$

Multiply numbers first.

$$16 = 4 \times h$$

Divide to get the variable all alone.

$$\frac{16}{4} = \frac{4h}{4}$$

$$4 = h$$

Check: $h = 4$

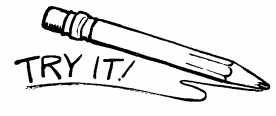
$$16 = \frac{1}{2} \times 8 \times h \quad \surd$$

$$16 = \frac{1}{2} \times 8 \times (\ )$$

$$16 = \frac{1}{2} \times 8 \times (4)$$

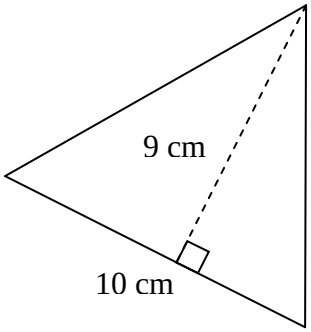
$$16 = \frac{1}{2} \times 32$$

$$16 = 16$$

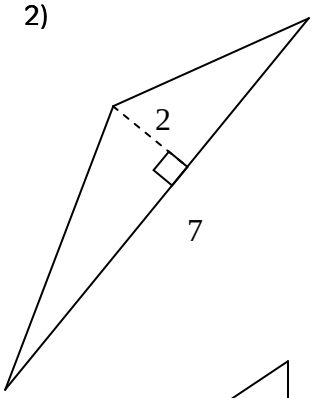


Find the area of the following triangles.

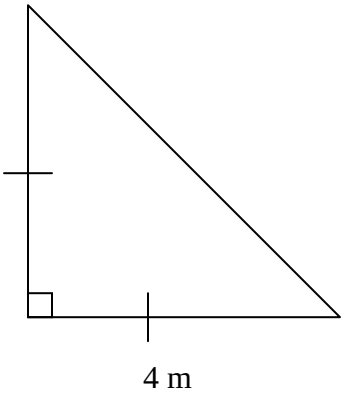
1)



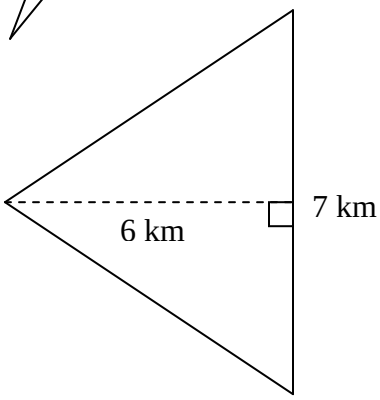
2)



3)

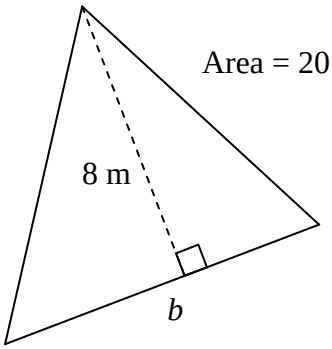


4)

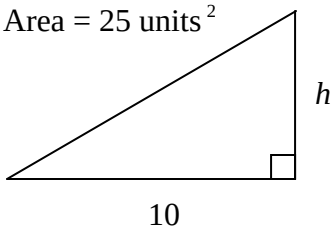


Find the missing base or height of the following triangles using the formula $A = \frac{1}{2}bh$.

5) Area = 20 m^2



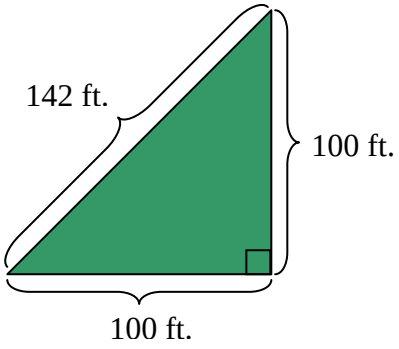
6) Area = 25 units^2



You talk with Jorge a few weeks later. He says that he was offered a different job at the airport. Now, they want him to edge the grass around the perimeter of the runways. He will earn two dollars for each foot of grass he edges. He wants you to help him figure out how much money he will make compared to what he earned cutting the grass.

Finding the perimeter of a triangle is the same as finding the perimeter of a quadrilateral. All we have to do is add up the lengths of all the sides.

In the triangle made by the runways, the perimeter is the sum of all the sides.



The perimeter is

$$100 + 100 + 142 = 342 \text{ ft.}$$

This means that Jorge will make

$342 \times \$2 = \$684$, for edging the grass.

Comparing \$684 with \$500 (the money he earnrd mowing),

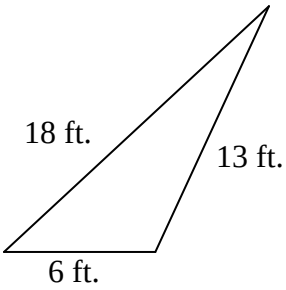
$$684 - 500 = \$184$$

Jorge can see that he will make \$184 more edging the grass than cutting it.

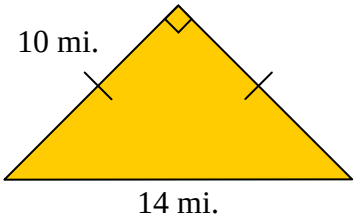
Example

Find the perimeter of the following triangles.

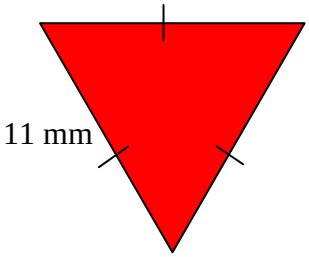
a)



b)



c)



Solution

To find the perimeter of each of these triangles, we have to add up all the sides.

a) We are given the length of all the sides in this triangle. The perimeter is

$$6 + 13 + 18 = 37 \text{ ft.}$$

b) We are only given two sides in the isosceles triangle. We know the third side is 10 mi., because the hash mark shows us that the unknown side is congruent to the side that is 10 mi.

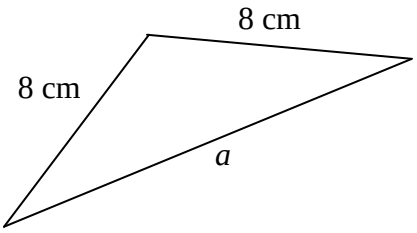
$$10 + 10 + 14 = 34 \text{ mi.}$$

c) We are only given one side of the equilateral triangle. All the sides are equal in an equilateral triangle, so all the sides are 11 mm.

$$11 + 11 + 11 = 33 \text{ mm}$$

Example

If the total perimeter of the triangle is 28 cm, find the length of the missing side.



Solution

Since the total perimeter is 28 cm, we know that the sum of the lengths of the sides is 28 cm. This means we can set up the equation,

$$8+8+a=28$$

$$16+a=28$$

$$-16 \qquad -16$$

$$a=12$$

Check: $a = 12$

$$8+8+a=28$$

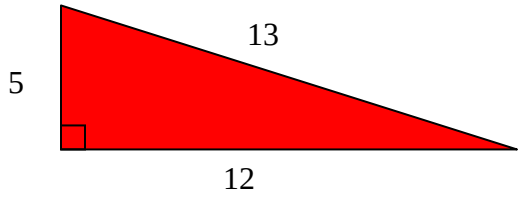
$$8+8+(\ )=28$$

$$8+8+(12)=28 \quad \surd$$

$$28=28$$

Find the area and perimeter of the following triangles.

7)



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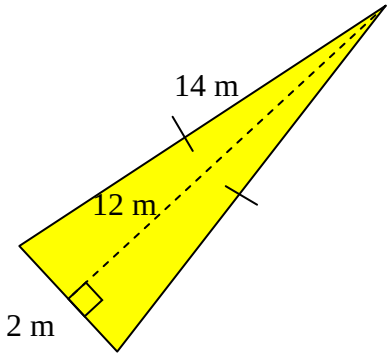
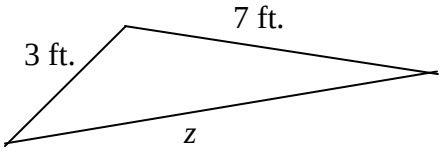


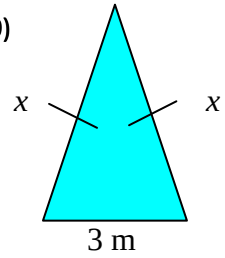
Given the following perimeters, find the lengths of the missing side(s) of the following triangles.

9)



Perimeter = 18 ft.

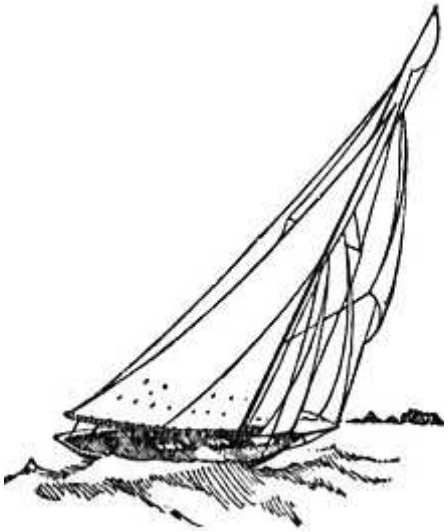
10)



Perimeter = 17 m.

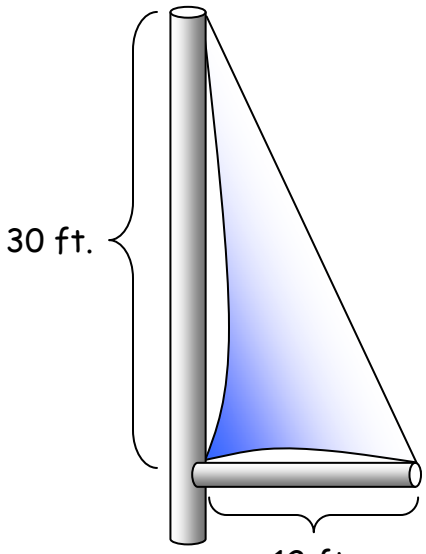
On Wednesdays, you volunteer at a local youth center where you mentor younger kids. One youth that you mentor, Ivette, needs some help. She explains.

"My Uncle Hernandez races sailboats. The boat he sails is called a 'sloop'. It is great for sailing upwind. He took me out on his sloop this summer, and I loved it so much that I wanted to build a scale model of it. The only problem is that my brother took the model out of the package and lost a few of the parts. Can you help me build new parts?"



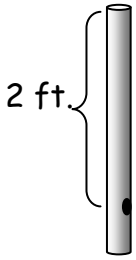
Sloop sailboat

She continues, "The main sail of my uncle's sloop has these measurements:" Ivette shows you a diagram she drew. "I only have one part of the main sail left."



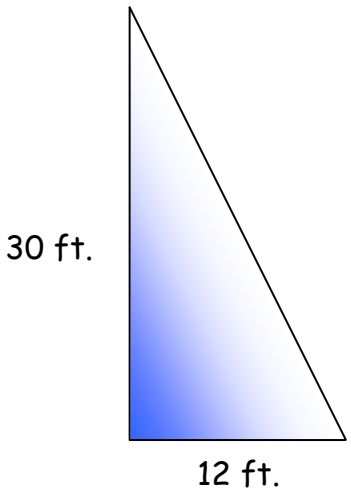
Uncle Hernandez's sloop

Ivette asks you, "How long should I make the other piece of my main sail?"



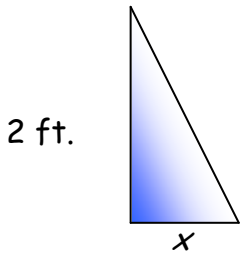
Ivette's sloop model

You realize that since the model has been made to scale, its sails and the actual sails of the sloop will form similar triangles. You draw a simplified version of Ivette's diagram.



Uncle Hernandez's sloop

The length of the bottom piece is not known, so you use a variable, x, to represent it.



Ivette's sloop model

Think Back

If two figures are similar, the lengths of their corresponding sides are proportional.

Triangles with three congruent angles are similar triangles. (This is only true for triangles.)

Since the figures are proportional, the ratio of the lengths of their sides will be equal. This means,

$$\frac{big\ bottom}{big\ side} = \frac{little\ bottom}{little\ side}$$

$$\frac{12}{30} = \frac{x}{2}$$

$\frac{12}{30} = \frac{x}{2}$ Cross multiply.

Think Back

When given a proportion, cross multiply, then divide.

$\frac{30x}{30} = \frac{24}{30}$ Divide each side by 30.

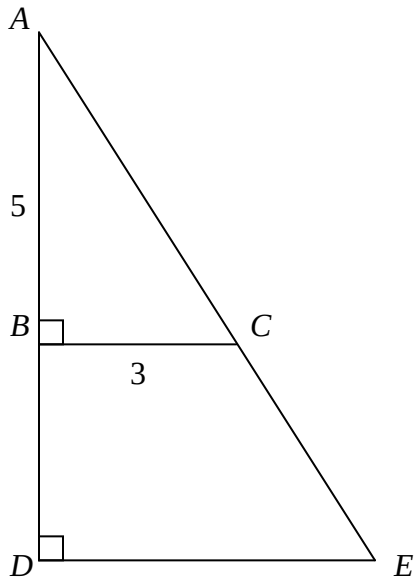
$x = 0.8$

You tell Ivette, "you should make a piece that sticks out eight-tenths of a foot."

"Thank you so much!" she exclaims. "You are the best!"

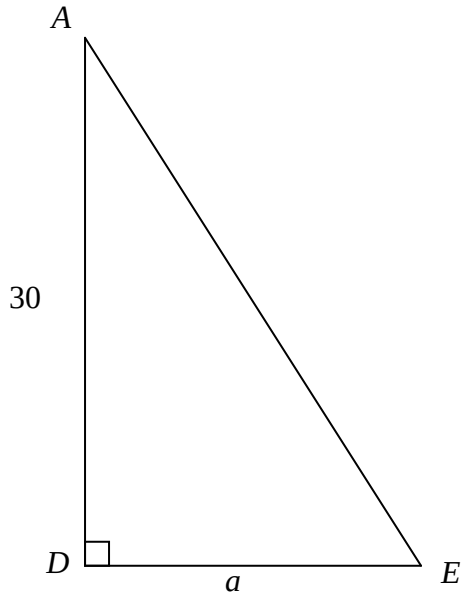
Example

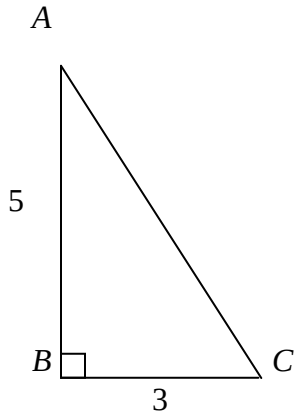
In this figure, *AD* = 30. What is the length of $\overline{DE}$?



Solution

How many triangles do you see in the diagram? You should see two: ΔABC and ΔADE. Let us redraw them below, with the measurements given. We will use a variable, *a*, to represent the side that we are trying to find, *DE*.





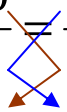
Next, we will set up our proportion. Note that there are many ways to set up proportions with similar figures. We will use the corresponding parts. That is to say,

$$\frac{big\ side}{little\ side} = \frac{big\ base}{little\ base}$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

Now substitute the lengths for each side.

$$\frac{30}{5} = \frac{a}{3}$$



Note: There are a few ways to solve this equation. Since cross multiplying always works, we will use this method. However, if you see a quicker way of solving this (and there is one), then feel free to use that. Just make sure you check your answer.

$$\frac{90}{5} = \frac{5a}{5}$$

$$18 = a$$

Check: $a = 18$

$$\frac{30}{5} = \frac{a}{3}$$

$$\frac{30}{5} = \frac{(\ )}{3}$$

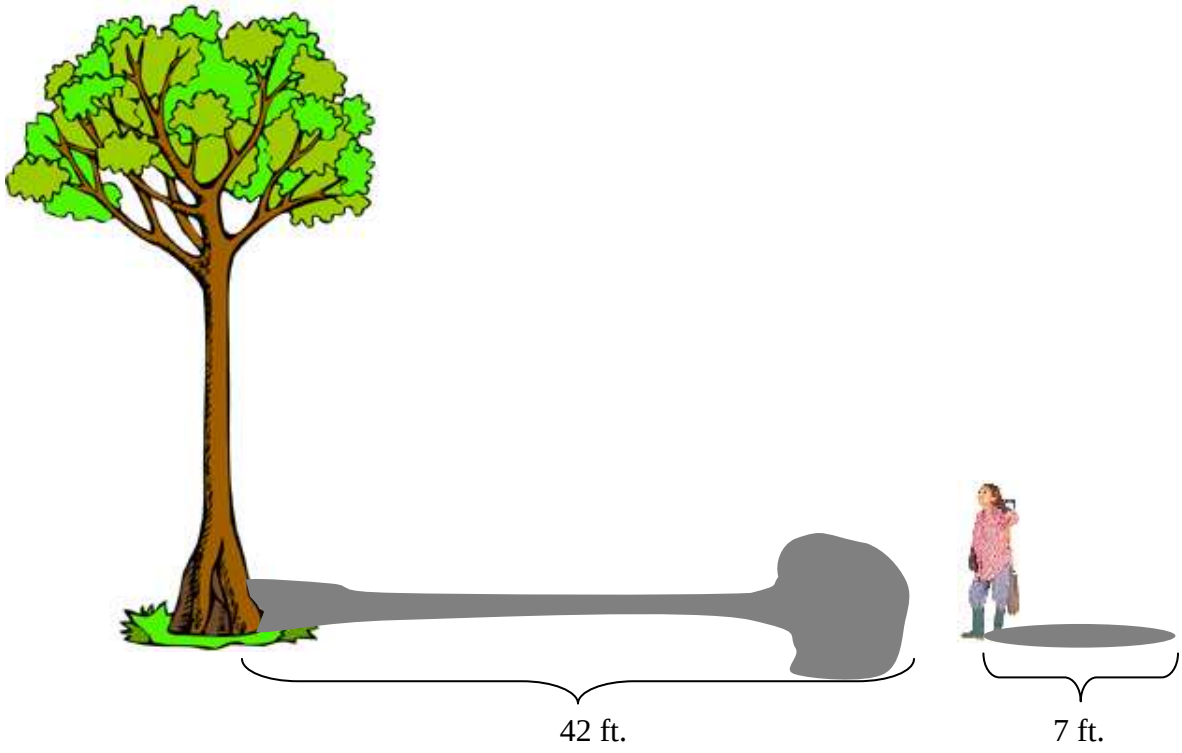
$$\frac{30}{5} = \frac{(18)}{3}$$

$$6 = \frac{18}{3}$$

$$6 = 6 \quad \surd$$

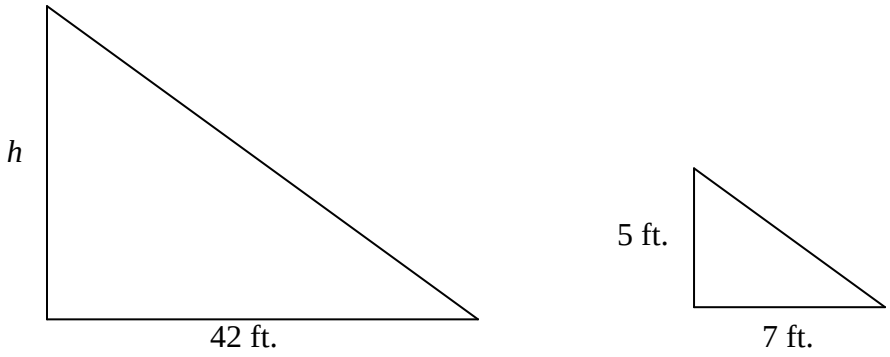
Example

Elena owns a landscaping business. One day, she gets a call from a customer who wants a tree cut down. She gets to the job site and needs to know how tall the tree is for billing reasons. She knows her own height is 5 feet. The sun is also casting the shadow of her and the tree. Find the height of the tree.



Solution

With a little imagination, we can redraw the diagram above using triangles. We label the information given and use a variable, *h*, to represent what we need to find.



These triangles are similar, so we can set up a proportion.

$$\frac{height\ of\ tree}{height\ of\ person} = \frac{shadow\ of\ tree}{shadow\ of\ person}$$

$$\frac{h}{5} = \frac{42}{7}$$

$$\frac{h}{5} \times \frac{42}{7}$$

$$\frac{7h}{7} = \frac{210}{7}$$

$$h = 30$$

Check: $h = 30$

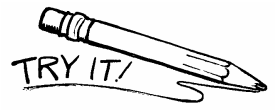
$$\frac{h}{5} = \frac{42}{7}$$

$$\frac{(\ )}{5} = \frac{42}{7}$$

$$\frac{(30)}{5} = \frac{42}{7}$$

$$6 = \frac{42}{7}$$

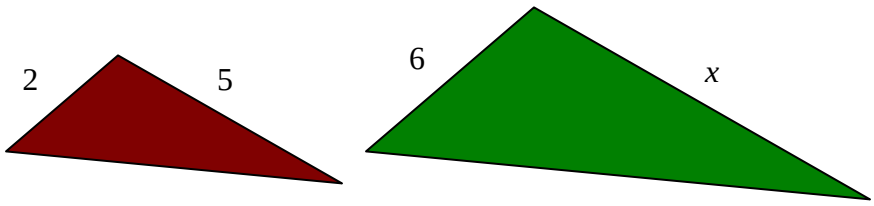
$$6 = 6 \quad \surd$$



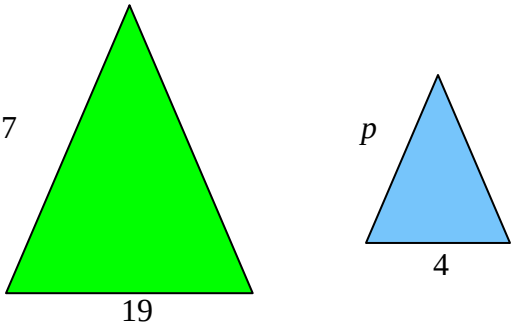
Given the similar triangles, use proportions to solve for the missing side.

(Round the the nearest tenth.)

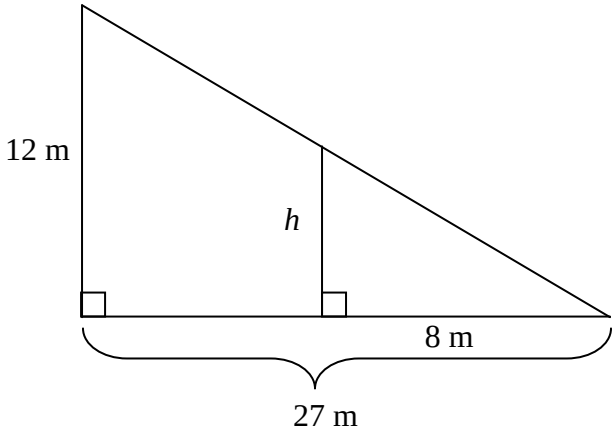
11)



12)



13)



Review

1. Highlight the definition of "diagonal."

2. Highlight the "Think Back" boxes.

3. Write one question that you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

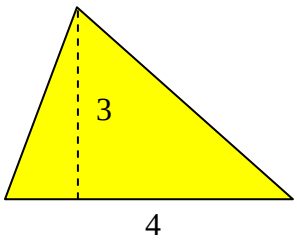
Math On the Move Lesson 20

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 20, Set A and Set B.

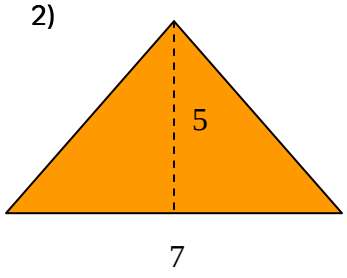
Set A

Use the formula $A = \frac{1}{2}bh$ to find the area of each triangle to the nearest tenth.

1)



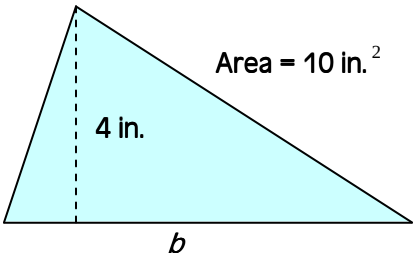
2)



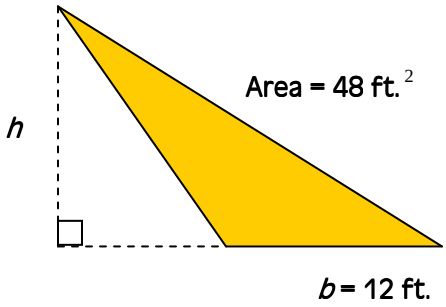
3) A triangle with a base of 17 feet, and a height of 15 feet

Use the formula, $A = \frac{1}{2}bh$ to find the missing base or height of each triangle.

4)



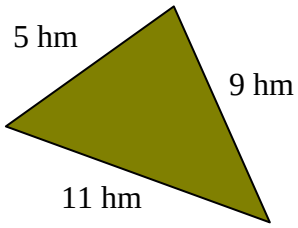
5)



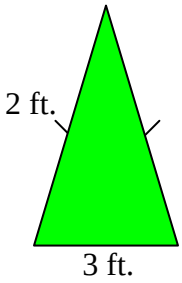
b = 12 ft.

Find the perimeter of each triangle.

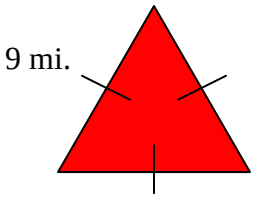
6)



7)

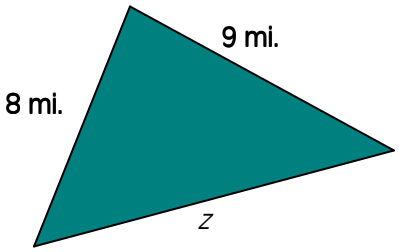


8)



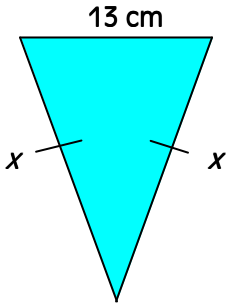
Given the perimeter, find the length of each missing side of each triangle.

9)



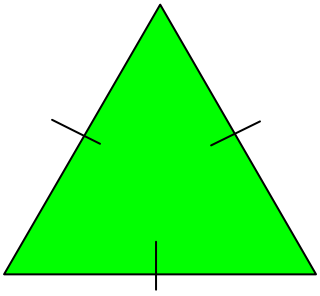
Perimeter = 27 mi.

10)



Perimeter = 41 cm

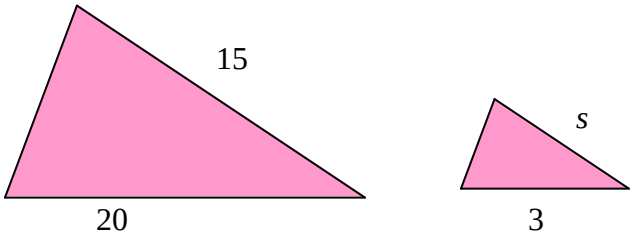
11)



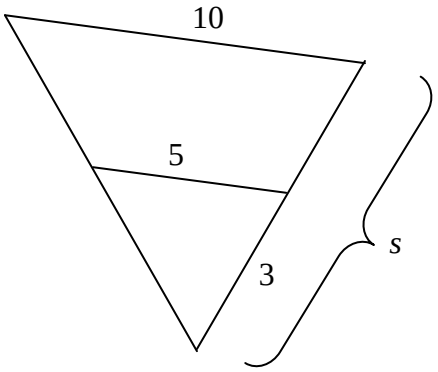
Perimeter = 78

Given similar triangles, find *s*.

12)



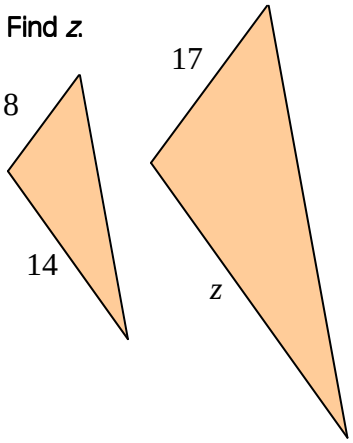
13)



Set B

1) Draw two polygons with corresponding congruent angles, that are <u>not</u> similar. (*Hint:* they will not be triangles.)
2) For the following problem:

Find *z*.



Melissa sets up the following proportion:

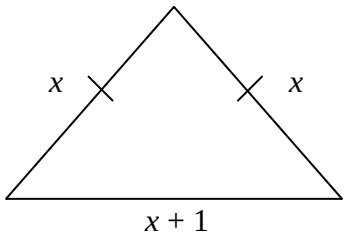
$$\frac{z}{14} = \frac{17}{8}$$

Hector sets up this instead:

$$\frac{17}{z} = \frac{8}{14}$$

Who is correct, Melissa, or Hector? Solve for *z* in both equations. What do you notice? Why do you think this is?

3) The perimeter of the following triangle is 16. Find the length of each side.



ANSWERS TO TRY IT

1) $A = 45\ cm^2$

2) $A = 7\ units^2$

3) $A = 8\ m^2$

4) $A = 21\ km^2$

5) $20 = \frac{1}{2} \cdot 8 \cdot b$

$b = 5\ m$

6) $25 = \frac{1}{2} \cdot 10 \cdot h$

$h = 5 units$

7) $A = 30\ units^2$

$P = 30\ units$

8) $A = 12\ m^2$

$P = 30\ m$

9) $z = 8\ ft.$

10) $2x + 3 = 17$

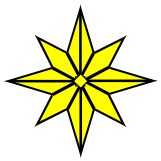
$x = 7\ m$

11) $x = 15 units$

12) $p = 1.5 units$

13) $h = 3.6\,m$

NOTES



End of Lesson 20