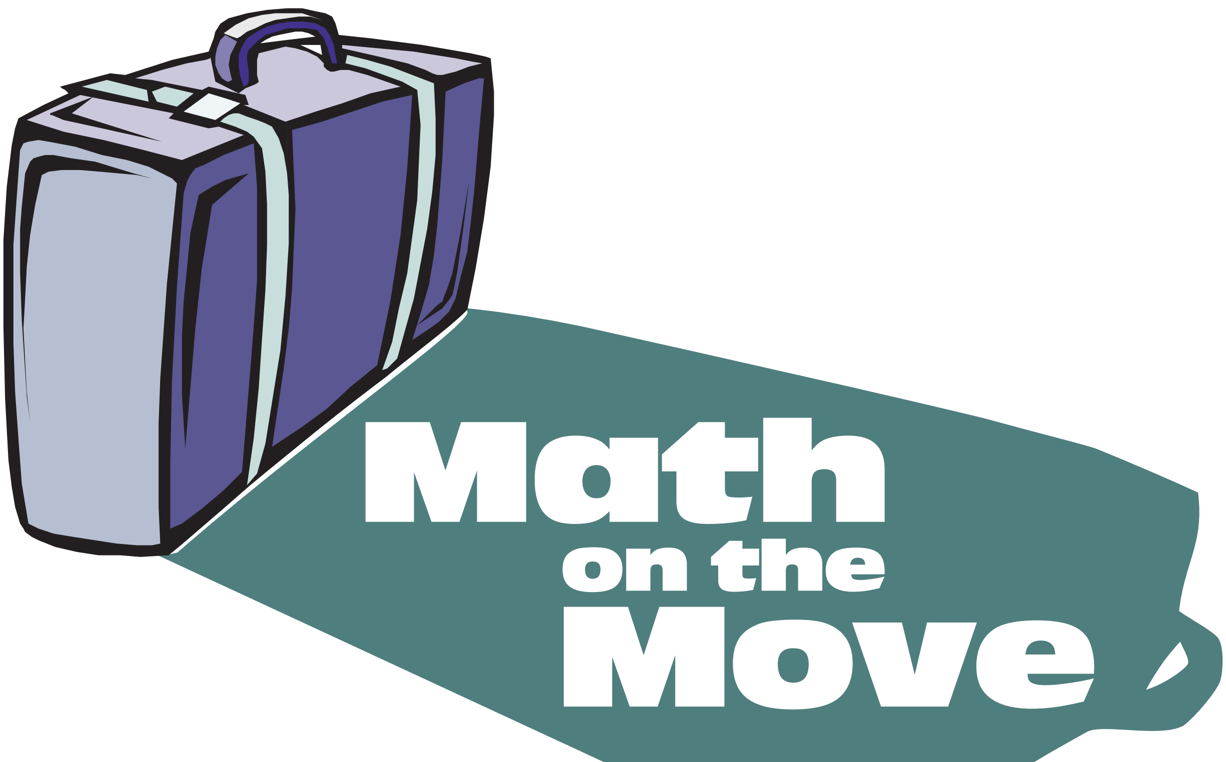
Student Name:	Contact Person Name:
Date:	Phone Number:



Lesson 19

Triangle Properties

Objectives

- Understand the definition of a triangle
- Distinguish between different types of triangles
- Use the Pythagorean Theorem to find the missing side in a right triangle

Authors:

Jason March, B.A.
Tim Wilson, B.A.

Editor:

Linda Shanks

Graphics:

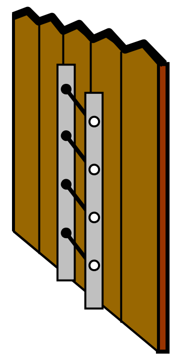
Tim Wilson
Jason March
Eva McKendry

National PASS Center
BOCES Geneseo Migrant Center
27 Lackawanna Avenue
Mount Morris, NY 14510
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass

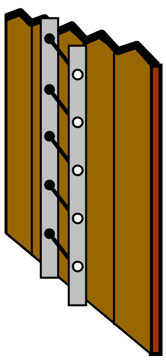
Genesee Valley
Board Of Cooperative Educational Services

Developed by the National PASS Center under the leadership of the National PASS Coordinating Committee with funding from the Region 20 Education Service Center, San Antonio, Texas, as part of the Mathematics Achievement = Success (MAS) Migrant Education Program Consortium Incentive project. In addition, program support from the Opportunities for Success for Out-of-School Youth (OSY) Migrant Education Program Consortium Incentive project under the leadership of the Kansas Migrant Education Program.

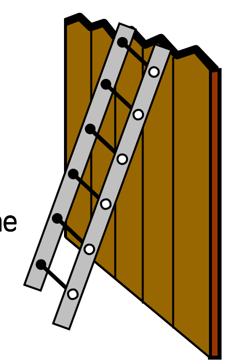
One day, you are playing football in your backyard with your friend, Julio. You believe that you can throw the ball farther than he can, so you challenge him to a throwing contest. Julio accepts your challenge and asks to throw first. Julio steps up and throws the ball so far that it goes over the fence that separates your yard from your neighbor's yard. Julio says, "Don't worry. I will use a ladder to climb the fence and get the ball. All I need is a ladder that is taller than the fence." You ask Julio why he needs a ladder taller than the fence, and he begins to explain.



"If I use a ladder shorter than the fence, it will not be tall enough for me to climb over it."



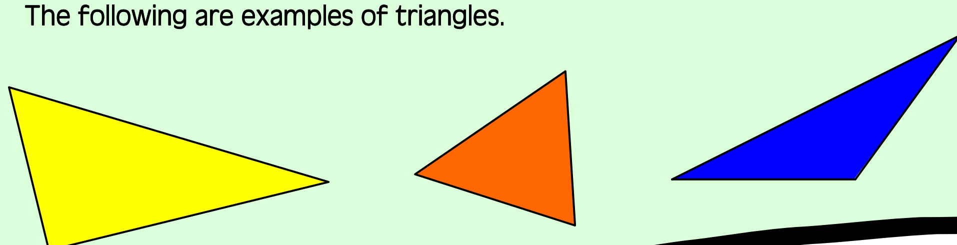
"If I use a ladder that is the same height as the fence, it will lie flat against the fence, and it will not be stable."



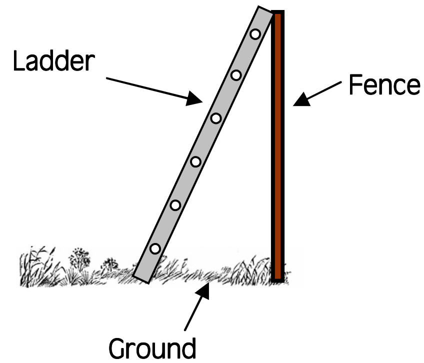
"I must use a ladder that is longer than the fence, so I can rest it at an <u>angle</u>."

Julio continues to explain, "If you look at the way the ladder leans up against the fence, it forms a **triangle**."

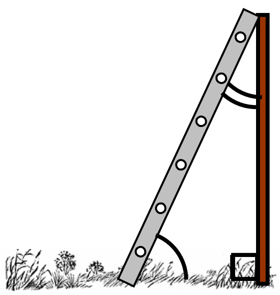
- A **triangle** is a polygon with three sides and three angles. The <u>prefix</u> "tri-" means three. So, a <u>triangle</u> is made up of three angles.
    - The following are examples of triangles.



Let's look at the side view of the ladder and the fence.



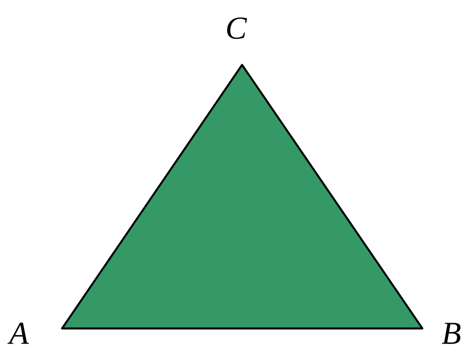
In the picture, we can see that the ladder leaning up against the fence forms a triangle. The three sides are made of the ground, the fence, and the ladder.



This triangle also has three angles. The three angles are formed between the ladder and the fence, the fence and the ground, and the ground and the ladder. When evaluating math problems, we assume that fences and walls are always built perpendicular to the ground. Thus, we use the small box symbol to show the right angle formed by the fence and the ground.

The triangle is the most basic polygon, with the least number of sides. It is impossible to make a two-sided polygon. We name a triangle by each of its <u>vertices</u>.

The following triangle is ΔABC. We use the symbol Δ to represent a triangle.



ΔABC has three sides: $\overline{AB}, \overline{BC},$ and $\overline{AC}$.

ΔABC has three angles: $\angle A, \angle B,$ and $\angle C$

ΔABC has three <u>vertices</u>: A, B, and C

Think Back

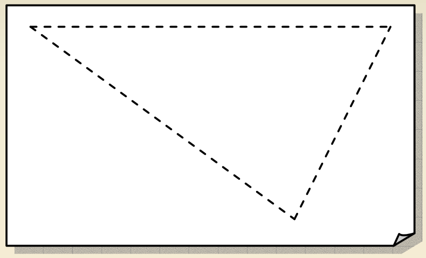
The word "vertex" was used for angles. "Vertex" is used in polygons for the points where sides meet and form angles. The vertex is a corner. "Vertices" is the word we use for more than one "vertex".

Triangles have some special properties.

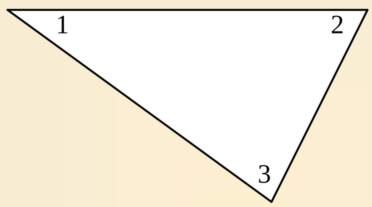
The sum of the angles in a triangle is $180°$.

Find out for yourself!

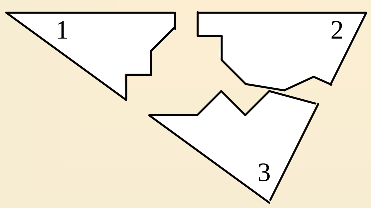
Step 1: Get a blank piece of paper. Use a straightedge and draw a triangle on that sheet of paper.



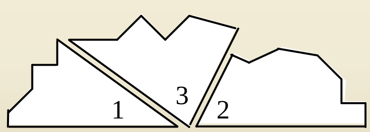
Step 2: Cut out your triangle, and number each vertex with a 1, 2, and 3.



Step 3: Cut or rip the three corners from the triangle.



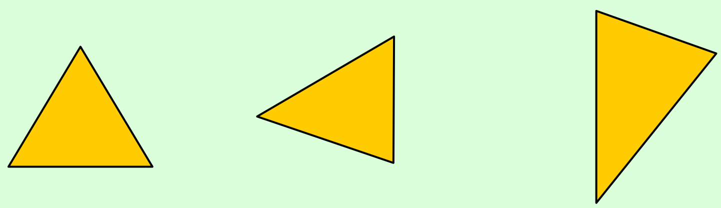
Step 4: Line up the three angles with their numbers pointing towards the middle.



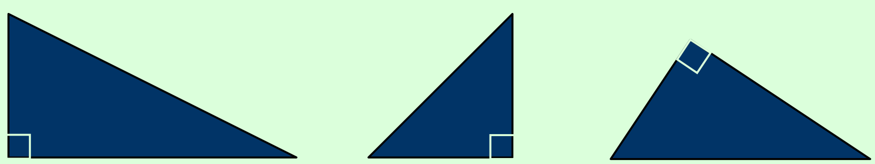
What do you notice about the way the three angles line up?

The sum of the angles is $180°$ in *every* triangle. Triangles can be classified by the types of angles they have. There are three types of triangles defined by their angles: **acute triangle**, **right triangle**, and **obtuse triangle**.

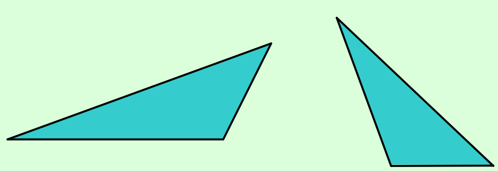
- An **acute triangle** is a triangle where *every* angle is an acute angle. Each angle is <u>less than</u> $90°$.
    - The following are examples of acute triangles.



- A **right triangle** is a triangle that has *one* right angle. One angle is <u>exactly</u> $90°$.
    - The following are examples of right triangles.



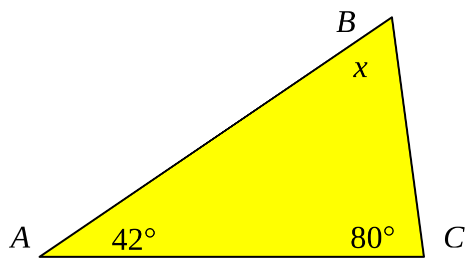
- An **obtuse triangle** is a triangle that has *one* obtuse angle. One angle is <u>between</u> $90°$ and $180°$.
    - The following are examples of obtuse triangles.



Right triangles can only have one right angle. If they had more than one, the sum of the angles would be greater than $180°$. *The same is true for obtuse triangles.*

Example

Find the missing angle in the triangle, and define the triangle as acute, right or obtuse.



Solution

The sum of the angles in every triangle is $180°$. The two angles we have are $42°$ and $80°$.

We know that $(m\angle A + m\angle B + m\angle C = 180°)$, so

$$42° + 80° + x = 180°.$$

Now we can solve this problem as an algebraic equation. First, combine similar terms.

$$42° + 80° + x = 180°$$

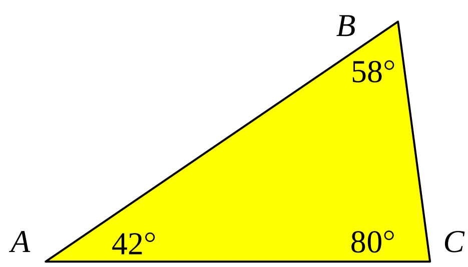
$$122° + x = 180°$$

Now we want to get the variable by itself.

$$\begin{array}{rcl} 122° + x & = & 180° \\ \underline{-122} & & \underline{-122} \end{array}$$

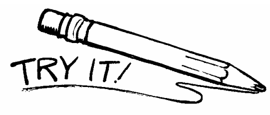
$$x = 58°$$

So, the missing angle is $58°$.



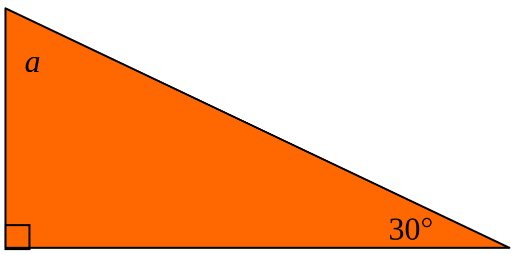
Each angle is less than $90°$, so ΔABC is an acute triangle.

Let's get some practice on our own.

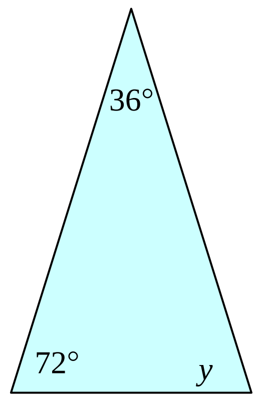


1. Find the measure of the missing angle in the given triangles. Then, classify the triangles as acute, right or obtuse.

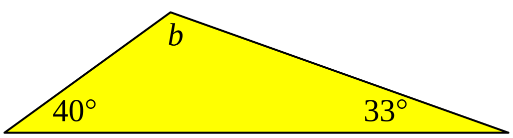
a)



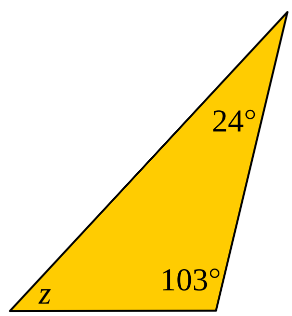
b)



c)



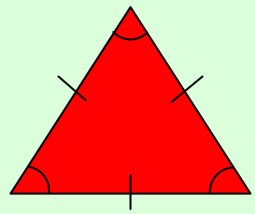
d)



Excellent! Now we need to classify triangles by the length of their sides. There are three ways to classify a triangle by the length of its sides: **equilateral**, **isosceles**, and **scalene**.

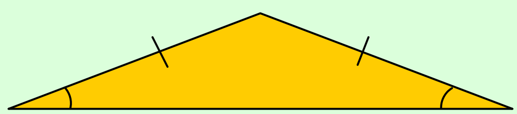
- An **equilateral triangle** is a triangle with all sides having equal measure. Notice that it sounds as if the word "equal" is in the word equilateral.
  - o All the angles in an equilateral triangle have equal measure.

The following is an example of an equilateral triangle.



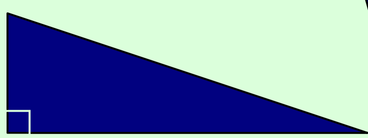
- An **isosceles triangle** is a triangle with two sides measuring the same length. Remember that isosceles trapezoids have two sides with the same length.
  - o Two of the angles in an isosceles triangle have the same measure.

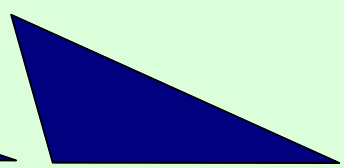
The following is an example of an isosceles triangle.



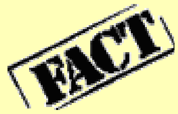
- A **scalene triangle** is a triangle with no sides measuring the same length. Any triangle that is not equilateral or isosceles is a scalene triangle.

The following are examples of scalene triangles.





Notice that one triangle is scalene and has a right angle in it. A triangle can be defined by its sides, as well as by its angles.

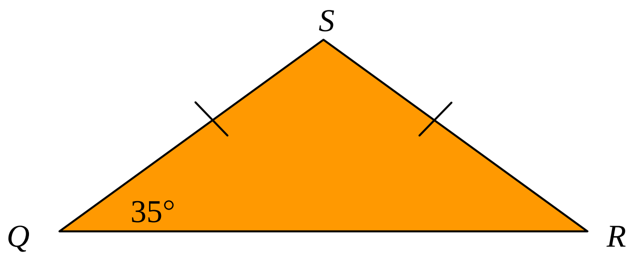


All three angles in an equilateral triangle have the same measure. If the angles add up to $180°$ *and each angle is the same measure, then each angle must be* $180° \div 3 = 60°$.

In an isosceles triangle, the angles opposite the sides of equal length are congruent.

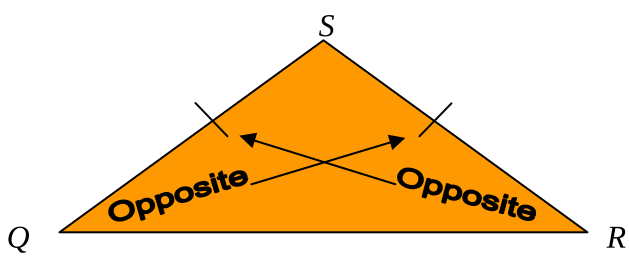
Example

Find the missing angles, and classify the triangle in as many ways as possible.

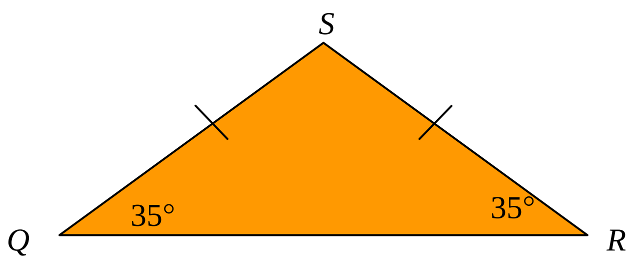


Solution

The first thing we know is that ΔQRS is an isosceles triangle because two sides have equal measure. We must use the fact that angles opposite sides of equal length are congruent.



Thus, we know that $\angle Q \cong \angle R$ which means that $m\angle Q = m\angle R = 35°$



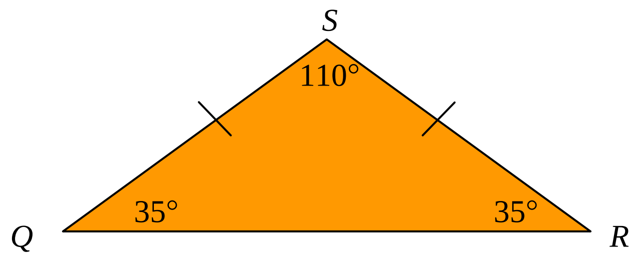
To find the last angle, we will use the fact that $m\angle Q + m\angle R + m\angle S = 180°$.

$$35° + 35° + m\measuredangle S = 180°$$

$$70° + m\measuredangle S = 180°$$

$$\underline{-70°} \qquad \underline{-70°}$$

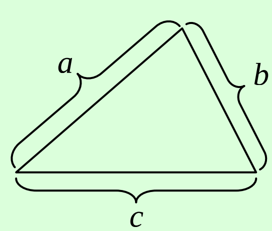
$$m\measuredangle S = 110°$$



ΔQRS is an obtuse, isosceles triangle with $m\measuredangle Q = m\measuredangle R = 35°$, and $m\measuredangle S = 110°$.

Another interesting thing about triangles is the **triangle inequality**.

- The **triangle inequality** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. More specifically, the sum of the two shorter sides is greater than the longest side.

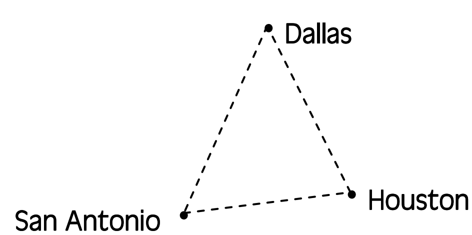


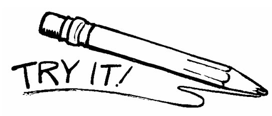
$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

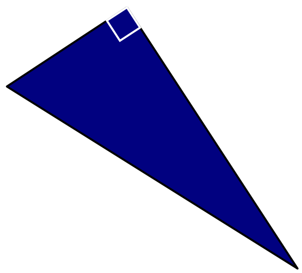
Think of the triangle inequality this way: suppose we want to travel between three cities (Houston, Dallas, and San Antonio). The shortest distance between two cities is a straight path between them. So, it is a shorter distance to travel directly between San Antonio and Houston as opposed to going from San Antonio to Dallas, and then from Dallas to Houston.



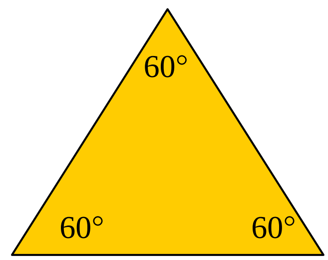


2. Classify the following triangles by their angles and their sides.

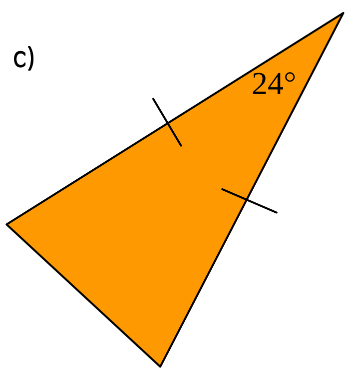
a)



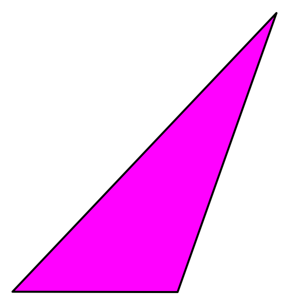
b)



c)

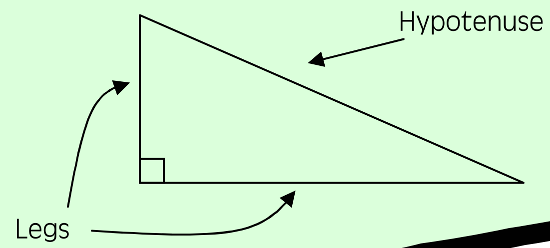


d)



Now that we have discussed all the types of triangles, we will focus on special properties of right triangles. Every triangle has three sides, but right triangles have special names for the three sides. The two shorter sides are called the **legs**, and the longest side is called the **hypotenuse**.

- The **legs** of a right triangle are the sides adjacent to the right angle.
- The **hypotenuse** of a right triangle is the side opposite the right angle. It is the longest side of the triangle.



FACT

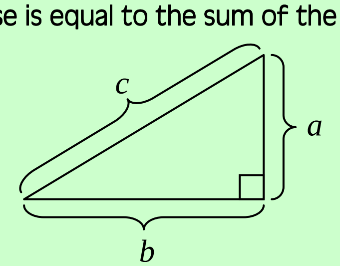
We use the word adjacent to talk about objects that are next to each other. Sides that are adjacent to an angle are the sides that meet to form that angle.

FACT

The longest side of a triangle is always opposite the largest angle. The largest angle in a right triangle is the right angle. So, the longest side of a right triangle is the hypotenuse.

One important property of right triangles is the **Pythagorean Theorem**.

- The **Pythagorean Theorem** states the following: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs,



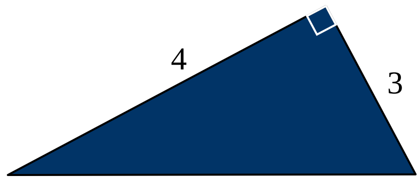
$$a^2 + b^2 = c^2$$

where a and b represent the legs of the triangle, and c represents the hypotenuse.

Let's use this theorem in the next example.

Example

Find the length of the hypotenuse of the following triangle.



Solution

We are given a right triangle and the lengths of both legs. We will use the Pythagorean Theorem $\left(a^2 + b^2 = c^2\right)$ to find the third side of the triangle. Since we were given the two legs, we know what *a* and *b* are.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \end{aligned}$$

The formula has given us the length of the hypotenuse squared $\left(c^2\right)$. However, we want *c,* not c^2. We need to take the **square root** of both sides.

- The **square root** of a number is any number that, when multiplied by itself, gives you the original number.
    - Just as subtraction is the opposite of addition and division is the opposite of multiplication, exponents have an opposite operation. The opposite of squaring a number (raising a number to the second power) is calculating the square root of a number. The symbol for a square root is $\sqrt{\ }$

$7 \times 7 = 7^2 = 49$	$10 \times 10 = 10^2 = 100$	$x \cdot x = x^2$
$\sqrt{49} = 7$	$\sqrt{100} = 10$	$\sqrt{x^2} = x$

We were left with the equation $25 = c^2$. When we solve equations for a variable, any operation we perform on one side of the equal sign, must be performed on the other side of the equal sign.

$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

The length of the hypotenuse, then, is 5.

This problem worked out nicely because our answer is a whole number. If you did not know that $\sqrt{25} = 5$, you should become familiar with the list of perfect squares.

$\sqrt{1} = 1$	$\sqrt{36} = 6$	$\sqrt{121} = 11$
$\sqrt{4} = 2$	$\sqrt{49} = 7$	$\sqrt{144} = 12$
$\sqrt{9} = 3$	$\sqrt{64} = 8$	$\sqrt{169} = 13$
$\sqrt{16} = 4$	$\sqrt{81} = 9$	$\sqrt{196} = 14$
$\sqrt{25} = 5$	$\sqrt{100} = 10$	$\sqrt{225} = 15$

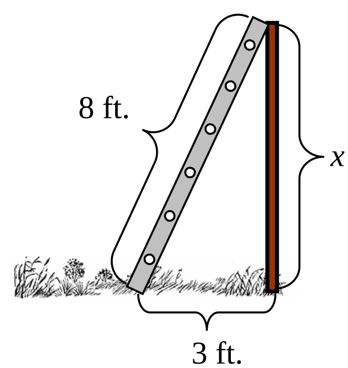
> **FACT**
>
> *A perfect square is a whole number whose square root is a whole number.*

Let's try one more together.

Example

You and your friend Julio successfully climbed the fence using the ladder. You find that you used a ladder that was 8 feet long. The distance you placed the base of the ladder from the fence was 3 feet. How tall was the fence?

(Round your answer to the nearest tenth)



Solution

To solve this problem, remember that this is a right triangle. The right angle is formed by the fence and the ground, so the hypotenuse is the ladder. We know the measure of two sides in the right triangle, so we can use the Pythagorean Theorem. 3 ft. is the value of a leg; the hypotenuse, *c*, is 8 ft.

$$a^2 + b^2 = c^2$$
$$3^2 + b^2 = 8^2$$
$$\begin{array}{rcl} 9 + b^2 & = & 64 \\ \underline{-9\quad} & & \underline{-9} \\ b^2 & = & 55 \end{array}$$

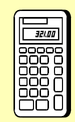
As shown before, we need to take the square root of both sides. But wait, 55 is not a perfect square! We need to use a calculator in order to solve this problem.

$$\sqrt{b^2} = \sqrt{55}$$
$$b = 7.4161985$$

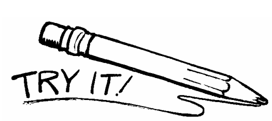
The question also said to round to the nearest tenth.

$$b \approx 7.4$$

The last thing we need to do is label the units. The ladder and the ground were measured in feet, so the fence is 7.4 feet tall.

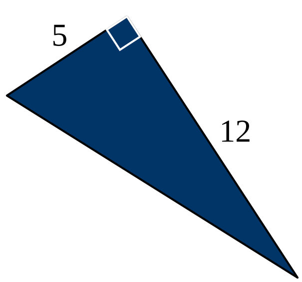
> ***Calculator Tip***
>
> 
>
>
> *To find the square root of a number, enter the number into the calculator, then press the square root button* √.

Try these problems on your own.

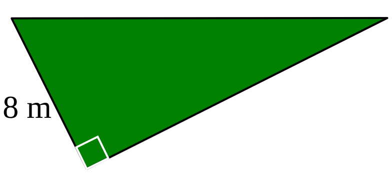


3. Identify the legs and hypotenuse of the triangles. Then find the missing side of the triangle. (Round your answer to the nearest hundredth when necessary.)

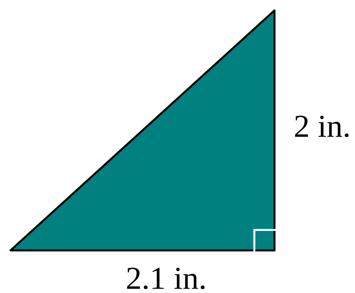
a)



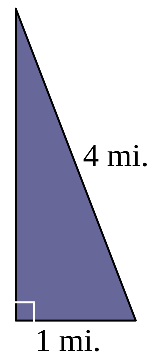
b)



c)



d)



Review

1. Highlight the following definitions:
   a. triangle
   b. acute triangle
   c. right triangle
   d. obtuse triangle
   e. equilateral triangle
   f. isosceles triangle
   g. scalene triangle
   h. triangle inequality
   i. legs
   j. hypotenuse
   k. Pythagorean Theorem
   l. square root

2. Highlight the "Fact" boxes.

3. Write one question that you would like to ask your mentor, or one new thing you learned in this lesson.

____

____

____

____

Practice Problems

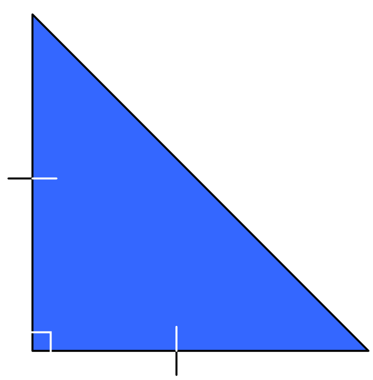
Math On the Move Lesson 19

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 19, Set A and Set B.

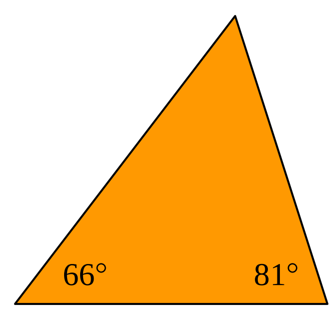
Set A

1. Find the missing angle(s) and classify the triangle in as many ways as possible.

a)



b)



2. State whether the following angle measures can make a triangle. If so, what type of triangle.

a) $36°, 60°, 70°$ b) $4°, 106°, 70°$ c) $50°, 50°, 90°$

d) $30°, 60°, 90°$ e) $112°, 34°, 34°$ f) $90°, 89°, 1°$

Set B

1. You can create a right isosceles triangle. Can you create a right equilateral triangle? Why or why not?

2. *True or False*: In a right triangle, the two acute angles are complementary. How do you know?

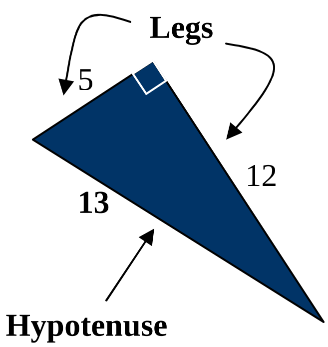
3. Dominick is standing 8000 ft. from the local airport. There is a plane circling 6000 ft. directly above the airport. How far is Dominick from the plane? (*Hint*: First, Draw a picture. Then, try to solve this using mental math. Imagine the distances are 8 ft. and 6 ft., and add three zeros to your solution. Check your answer with a calculator.)

ANSWERS TO TRY IT

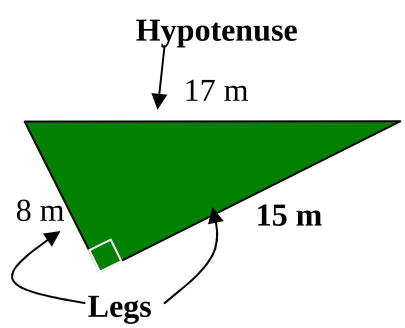
1. a) $60°$ right triangle
   b) $72°$ acute triangle
   c) $107°$ obtuse triangle
   d) $53°$ obtuse triangle

2. a) Right, scalene
   b) Acute, equilateral
   c) Acute, isosceles
   d) Obtuse, scalene

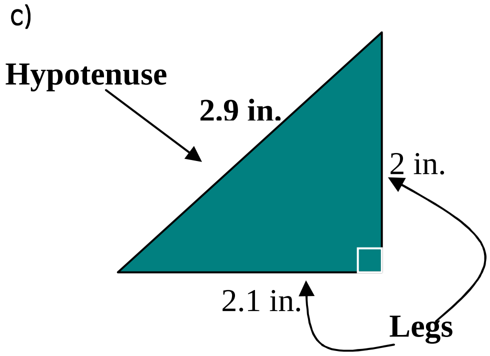
3. a)



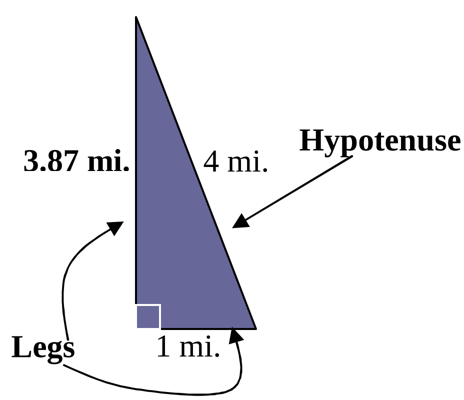
b)



c)



d)



NOTES

End of Lesson 19