Student Name: ______________________	Contact Person Name: ______________________
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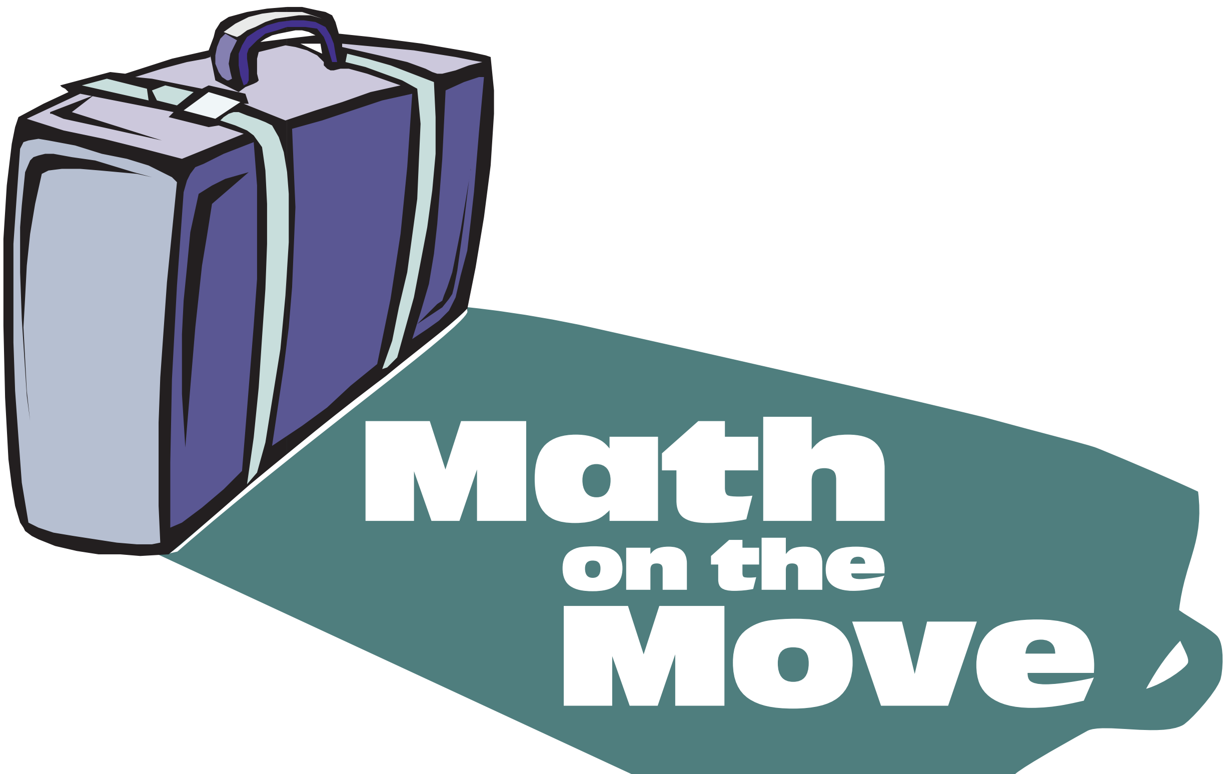

Lesson 15

Rates and Ratios

Objectives

- Understand what a rate and a ratio are
- Solve word problems that involve rates and ratios

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You and your friend, María, own a car wash. You want to be the fastest car wash in town, so you name your car wash Speedy Wash. Speedy Wash can wash and wax a car in 5 minutes. There are two other car washes in town. Jet Wash can wash and wax 10 cars per hour, and Laser Wash can wash and wax 3 cars every 20 minutes. How do you know if you have the fastest car wash?

You know how fast you and María can wash one car, but the other car washes use **rates** to describe how fast they wash cars.

- A **rate** is a comparison of two measures with different units.
  - In the United States, the speed of a car is measured in miles <u>per</u> hour. We're comparing miles (distance) to hours (time).

FACT

The word per means, "for every". Miles per hour means miles for every hour.

In order to figure out which car wash is the fastest, we need to compare how fast they wash cars at similar rates. Think of it this way, if Speedy Wash can wash and wax a car in 5 minutes, how many cars can it wash and wax in one hour?

To figure that out, we have to figure out how many five minute intervals there are in one hour. One hour is equivalent to 60 minutes, so there are $60 \div 5 = 12$ five-minute intervals in one hour. If the Speedy Wash can wash and wax one car in each of those intervals, they could wash 12 cars per hour.

We just converted Speedy Wash's car wash/wax rate into cars per hour. Now we need to do the same for the Laser Wash.

The Laser Wash washes and waxes 3 cars every 20 minutes. We need to figure out how many twenty minute intervals are in one hour. There are $60 \div 20 = 3$ twenty minute intervals in one hour. If the Laser Wash can wash and wax 3 cars in each of those intervals, they could wash $3 \times 3 = 9$ cars per hour. The rate for Jet Wash is already in cars per hour – 10 cars per hour.

Now that all the rates are in cars per hour, we can compare them to see which is the fastest.

Speedy Wash	Jet Wash	Laser Wash
12 cars/hr.	10 cars/hr.	9 cars/hr.

The chart clearly shows that Speedy Wash can wax and wash the most cars in one hour.

The slash, "/", is used to represent the word per. We use the slash, "/", to represent division, and rates are often the quotient of $\frac{something}{time}$. *In the car wash problem the rate is* $\frac{cars}{time}$. *More specifically,* $\frac{cars}{hour}$.

Rates will depict the quotient of two quantities measured in different units. The most common rate is speed.

To measure the average speed of an object, you must take the total distance traveled by that object and divide it by the time it took to travel that distance. To be more specific,

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Example

Manuel and Fernando are planning a road trip from New York City to Miami, Florida. The distance between the two cities is about 1300 miles. If they wanted to make it to Miami in 20 hours, what would their average speed have to be?

Solution

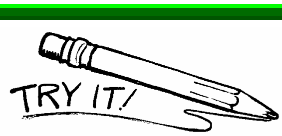
Average speed is found by dividing the total distance traveled by the total time it takes to travel that distance. So, if the distance between those two cities is 1300 miles, and they wanted to complete the trip in 20 hours, they would need to travel at a speed of

$$\frac{1300}{20} = 65.$$

We are not done with the problem, because we still need to label the speed with units. Well, we found the speed by dividing miles by hours. So, the units should be $\frac{\text{miles}}{\text{hour}}$. But in standard notation, we would write it as miles/hour (mi./hr.), or miles per hour (mph). Manuel and Fernando would have to travel at an average speed of 65 mph.

Whenever we want to find a rate, we must find the quotient of the two different units. This is similar to reducing the fraction by making the denominator a one. Remember that rates use the word "per", which means "for every". So, $\frac{1300 \text{ mi.}}{20 \text{ hr.}}$ reduces to $\frac{65 \text{ mi.}}{1 \text{ hr.}}$ which means the car goes 65 miles for every hour. As we can see, this is clearly the same as 65 miles per hour.

Try some rate problems on your own.



1. Solve the following rate problems:

a) Isabel works 40 hours a week as a dental hygienist. If she makes $980 each week, how much does she make per hour.

b) Ramón cuts grass for a landscaping company. He mows 6 lawns a day and works 5 days a week. If Ramón makes $750 a week, how much does he make per lawn?

The most common rate of measurement is speed. In the United States, we measure the speed of a car in miles per hour (mph). In most other countries, the speed of a car is measured in km/hr. Similar to comparing fractions, we must make sure that when we compare rates, they are in the same units. You will not be asked to convert from metric units to customary units, but suppose you were asked the following question.

Example

Car *A* is traveling 1 mi./min., and car *B* is traveling 55 mi./hr. Which car is traveling faster?

Solution

When we first look at this problem, we want to say that car *B* is going faster because $55 > 1$. However, we cannot say this because the two rates we have are in different units. So, we will convert 1 mi./min. into mi./hr.

To do this we need to look at the rate as a fraction.

$$1 \text{ mi./min.} = \frac{1 \text{ mi.}}{1 \text{ min.}}$$

All whole numbers can be written as a fraction with 1 in the denominator

If we multiply the fraction by the number 1, it does not change the value of the fraction. We need to multiply our fraction by something that will make the denominator turn into hours.

$$\frac{1\text{ mi.}}{1\text{ min.}}\left(\frac{60\text{ min.}}{1\text{ hr.}}\right)$$

There are 60 minutes in an hour, so $\frac{60\text{ min.}}{1\text{ hr.}} = 1$

If we multiply our fractions, we get

$$\frac{1\text{ mi.}}{1\ \cancel{\text{min}}.}\left(\frac{60\ \cancel{\text{min}}.}{1\text{ hr.}}\right) = \frac{60\text{ mi.}}{1\text{ hr.}}$$

Since the denominator is now 1, we can rewrite this as a rate.

$$\frac{60\text{ mi.}}{1\text{ hr.}} = 60\text{ mi./hr.}$$

Now we have both rates of speed written in the same unit. Car A is going 60 mi./hr. and car B is going 55 mi./hr. Thus, car A is going faster.

Let's try one more together.

Example

Pedro Martinez can throw a baseball 95 mph. How fast can he throw a baseball in ft./sec.?

(Round your answer to the nearest tenth.)

Solution

To solve this problem we will rewrite our rate as a fraction.

$$95\text{ mph} = \frac{95\text{ mi.}}{1\text{ hr.}}$$

First, we will change hours to seconds. There are 60 minutes in an hour and 60 seconds in a minute, so there are $60 \times 60 = 3600$ seconds in an hour.

$$\frac{95\text{ mi.}}{1\ \cancel{\text{hr}}.}\left(\frac{1\ \cancel{\text{hr}}.}{3600\text{ sec.}}\right) = \frac{95\text{ mi.}}{3600\text{ sec.}}$$

When we multiply fractions, we multiply the numerators, and then we multiply the denominators. Also, since the order that we multiply things does not matter, we can switch the denominators as we did above. We always set up our fraction so the units we do not want will cancel out. Next, we will convert miles to feet. There are 5280 ft in a mile.

$$\frac{95\ \cancel{\text{mi}}.}{3600\ \text{sec.}}\left(\frac{5280\ \text{ft.}}{1\ \cancel{\text{mi}}.}\right)=\frac{95\times 5280\ \text{ft.}}{3600\ \text{sec.}}$$

Again, we switch feet and miles so that the miles will cancel out. Now, if we multiply the numerator, we get

$$\frac{95\times 5280\ \text{ft.}}{3600\ \text{sec.}}=\frac{501,600\ \text{ft.}}{3600\ \text{sec.}}$$

In order for this to be an official rate, we must reduce the fraction so the denominator is one. In other words, we must divide 501,600 by 3600.

$$\frac{501,600\ \text{ft.}}{3600\ \text{sec.}}=\frac{139.\overline{3}\ \text{ft.}}{1\ \text{sec.}}\approx 139.3\ \text{ft./sec.}$$

So, Pedro Martinez throws a baseball 139.3 ft./sec.

This information may seem useless, but it can be interesting if we use it the right way. Pedro Martinez is a pitcher in baseball. He has to throw the ball from the mound to home plate. That's a distance of 60 ft. 6 in., or 60.5 ft. If Pedro throws a 95 mph ($139.\overline{3}$ ft./sec.) fastball, it would take the ball $\frac{60.5}{139.\overline{3}}\approx .434\ \text{sec.}$ to reach home plate. This means the batter has less than half of a second to react to the pitch!

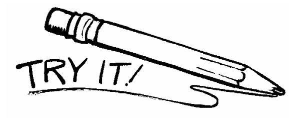
Think Back

We use the symbol "$\approx$" when we are rounding a value. The symbol means that two things are almost equal but not exactly equal. For example, $.99999\approx 1$.

FACT

Notice that we did not use the rounded answer to calculate the time it takes for the ball to reach home plate. If the repeating decimal will be a part of a further calculation, do not round until the end.

Try some converting problems on your own.



2. Solve the following rate problems:

a) Roselyn works as a general contractor. Her annual salary is $39,000 (Annual salary is how much she makes in one year). How much does she make per month?

b) Moises and Rich are having a race. Moises claims that he can run 22 ft./sec., and Rich says that he can run 264 in./sec. If they run a 100 yard race, who will win?

(*Hint*: convert the rates to the same unit.)

Similar to rates are **ratios**.

- A **ratio** is a comparison of numbers with the same units. The two numbers being compared cannot share any common factors (except the number 1). Ratios can be written the following ways.

$$\frac{2}{1} = 2:1 = 2 \text{ to } 1$$

If the ratio is written as a fraction, the fraction must be in simplest form. This is because the two numbers being compared cannot have any common factors (except 1).

Ratios are often used to compare the numbers of objects in a group.

Example

Mrs. Santiago's history class has 30 students in it – 12 boys and 18 girls. What is the ratio of boys to girls in her history class?

Solution

When finding the ratio, we must make sure we write it in the order the problem asks us. This problem asks for the ratio of boys to girls, so we write the number of boys first. Since we have done so much work with fractions, we will write the ratio as a fraction.

$$12 \text{ boys to } 18 \text{ girls} = 12:18 = \frac{12}{18}$$

This is good so far, but this fraction is not in simplest form. 12 and 18 share a common factor of 6, so we will reduce our fraction as follows.

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

The ratio of boys to girls is 2 to 3.

If you said that the ratio of boys to girls was 3 to 2, you would be wrong because that would mean there are more boys than girls.

Ratios can also help us determine how many objects are in each group.

Example

Carmela has a bag full of 90 marbles. There are only two different colors of marbles in the bag – red and blue. The ratio of red : blue is 3 : 2. How many red and blue marbles are in the bag?

Solution

The nice thing about ratios is that all multiples of the ratio are equivalent. 3 : 2 is equivalent to 30 : 20 and 300 : 200. These are all equivalent forms of 3 : 2.

$$3 : 2 = \frac{3}{2} = \frac{30}{20} = \frac{300}{200} = \frac{3x}{2x}\ldots$$

Now that we have established some of the equivalent forms of 3 : 2 we can solve the problem.

We know that the total number of marbles is 90. If we add the number of marbles that are in the ratio of 3 : 2, they should equal 90. Since we do not know the exact numbers, we must add the two ratio numbers with the variables and set that equal to 90.

$$3x + 2x = 90$$

This works because $3x : 2x$ is in the ratio 3 : 2. Now we solve this as if it were any other equation.

$$3x + 2x = 90$$

$$\frac{5x}{5} = \frac{90}{5}$$

$$x = 18$$

We may have found x, but we are not done. Remember that red : blue = 3 : 2 and $3x : 2x = 3 : 2$. So, red : blue = $3x : 2x$. This means that the number of red marbles is $3x$ and the number of blue marbles is $2x$.

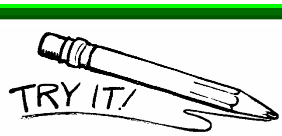
$$x = 18$$
$$3x = 54$$
$$2x = 36$$

So, there are 54 red marbles and 36 blue marbles. Let's check our answer.

$$54 + 36 = 90 \qquad \frac{54 \div 18}{36 \div 18} = \frac{3}{2}$$
$$90 = 90$$

The two numbers add up to 90 and they are in a ratio of 3 : 2, so our answer works.

Try some ratio problems on your own.



3. Solve the following ratio problems:

a) In Mr. Alvarez's math class, there are 10 boys and 8 girls. What is the ratio of boys to girls?

b) Isabel has a bag of chocolate and caramel candies. If there are 150 total candies in the bag and 100 of them are chocolate, what is the ratio of caramel to chocolate?

c) There are 165 seniors at Kennedy High School. If the ratio of boys to girls is 5 to 6, how many boys and girls are there in the senior class?

d) At Hurtz automotive, they rent two types of cars – SUVs and sedans. If the ratio of SUVs to sedans is $\frac{3}{7}$ and there are 250 total cars, how many SUVs and sedans are there?

Review

1. Highlight the following definitions:
    a. rate
    b. ratio

2. Highlight the “Fact” boxes.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 15

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 15, Set A and Set B.

Set A

1. Rewrite the following ratios in fraction form.

a) 1 : 2 b) 2 : 3 c) 3 : 8 d) 2 : 5 e) 17 : 1 f) 13 : 15

2. Write the following quotients as rates.

a) $\frac{90 \text{ mi.}}{30 \text{ hr.}}$ b) $\frac{550 \text{ ft.}}{11 \text{ sec.}}$ c) $\frac{3550 \text{ dollars}}{10 \text{ weeks}}$ d) $\frac{92 \text{ dollars}}{8 \text{ hrs.}}$

Set B

1. Curtis Martin can run a 40-yd. dash in 4.4 seconds. How fast did he run in mph. (*Hint*: there are 1760 yards in a mile. Start with a fraction and convert each unit, step by step.) Round to the nearest tenth.

2. Juliana has a bag full of red, white, and blue marbles. There are 175 marbles in the bag. The ratio of red:white is 1:2, and the ratio of white:blue is 1:2. How many of each color are in the bag. (*Hint*: what is the ratio of red:blue? What is the ratio of red:white:blue?)

ANSWERS TO TRY IT

1a) $\dfrac{\$980}{40\text{ hr.}} = \$24.50/\text{hr.}$

b) $5 \times 6 = 30$ lawns a week $\dfrac{\$750}{30\text{ lawns}} = \$25/\text{lawn}$

2a) $\$39,000/\text{year} = \dfrac{\$39,000}{1\text{ year}}\left(\dfrac{1\text{ year}}{12\text{ months}}\right) = \dfrac{\$39,000}{12\text{ months}} = \$3,250/\text{month}$

b) $264\text{ in./sec.} = \dfrac{264\text{ in.}}{1\text{ sec.}}\left(\dfrac{1\text{ ft.}}{12\text{ in.}}\right) = \dfrac{264\text{ in.}}{12\text{ sec.}} = 22\text{ ft./sec.}$ They both run the same speed, so they will tie.

3a) 5 to 4 or 5 : 4 or $\dfrac{5}{4}$

b) 150 – 100 = 50 caramels

$$\frac{50}{100} = \frac{1}{2} = 1 : 2 = 1 \text{ to } 2$$

c)
$$\begin{aligned} 5x + 6x &= 165 \\ 11x &= 165 \\ x &= 15 \end{aligned}$$

boys = $5x = 75$
girls = $6x = 90$

d)
$$\begin{aligned} 3x + 7x &= 250 \\ 10x &= 250 \\ x &= 25 \end{aligned}$$

SUVs = $3x = 75$
sedans = $7x = 175$

NOTES

End of Lesson 15