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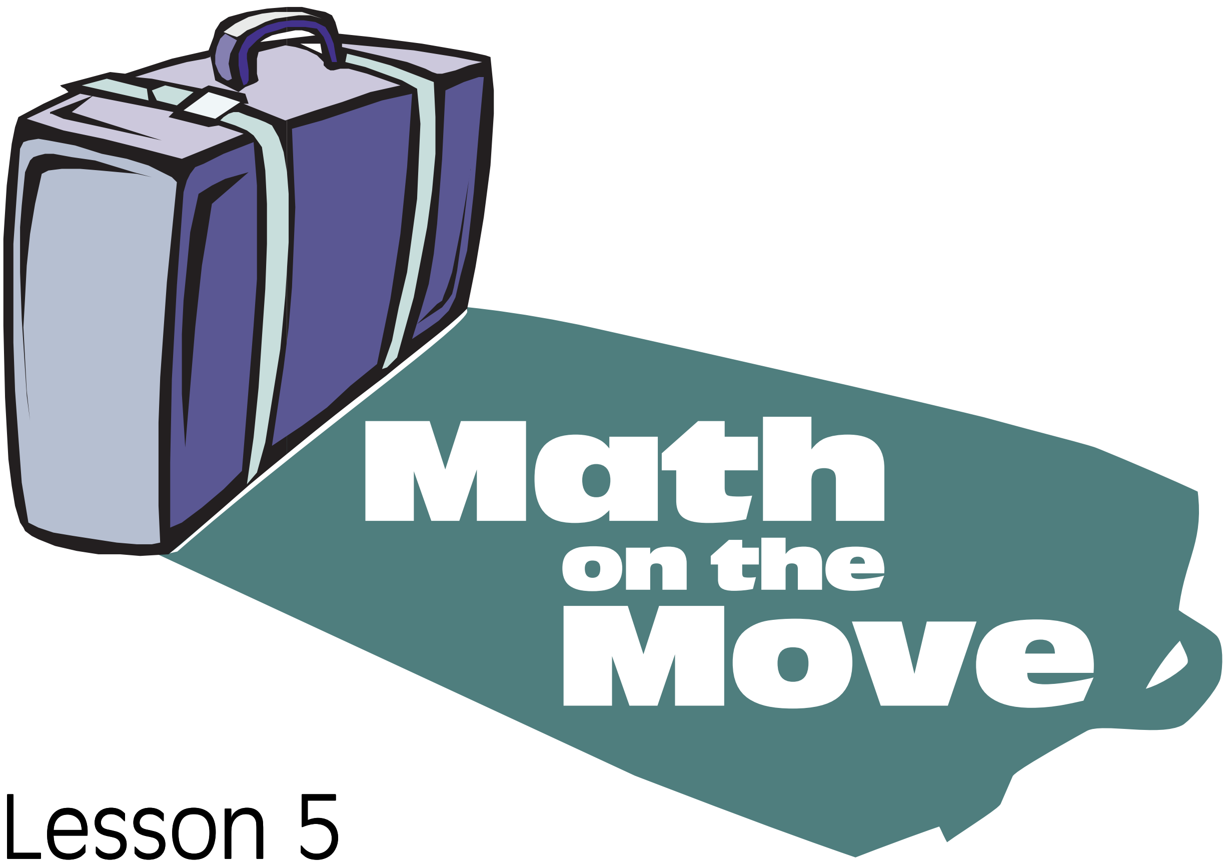

Lesson 5

Introduction to Fractions

Objectives

- Understand what a fraction represents
- Find equivalent fractions

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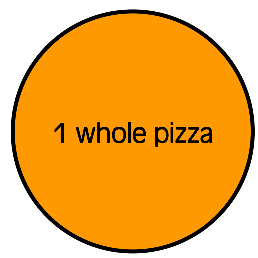
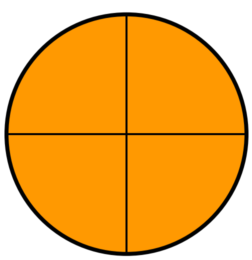
Imagine you are a pizza maker. One night, a family of four walks into your shop and orders a pizza, but they ask you to cut it so that each family member gets a slice that is the same size as every other slice. Right after them, a family of five walks in and places the same kind of order, so you cut their pizza into five equal slices and give each family member one slice. Then, a family of ten walks in, and makes the same order! Which family is going to get the biggest slice of pizza for each family member?

Hopefully, you understand that even though the family of ten got ten slices of pizza, a slice of their pizza is much smaller than a slice of pizza from the family of five, and smaller still than a slice from the family of four. Each member of the family of four is eating more pizza than the members of the family of five and ten.

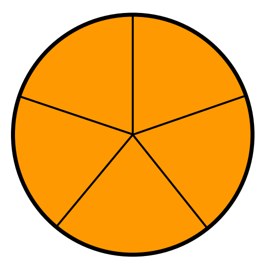
We can show this using math and pictures!
Pizza is usually round, so we will let a circle represent one whole pizza.



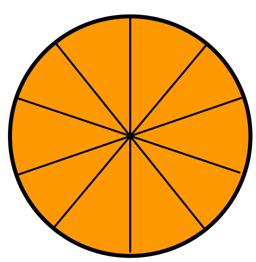
If we wanted to represent the pizza for the family of four, we must divide one whole pizza into four equal slices.



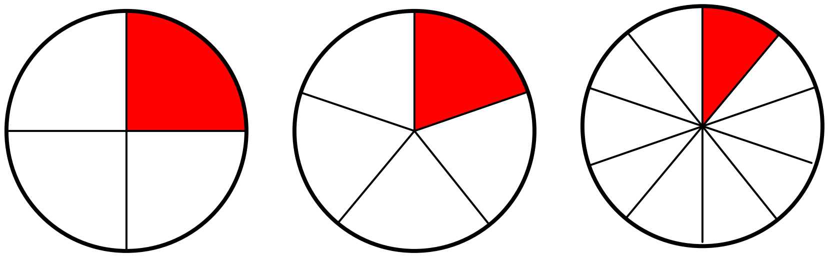
To represent the pizza for the family of five, we divide one whole pizza into five equal slices.



To represent the pizza for the family of ten, we divide one whole pizza into ten equal slices.



Remember that the question asked which family was going to get the biggest slice for each family member. So, we must compare single slices from each pizza.



If we compare the size of the shaded regions in the picture above, we see that the pizza cut into four equal slices has the largest shaded region. This means that the family of four will get the largest slice for each family member.

These pictures can be related to **fractions**, because we are focusing on a part of the whole.

- A **fraction** is the quotient of two numbers, *a* and *b*. A fraction is written as $\frac{a}{b}$, and it means $a \div b$.

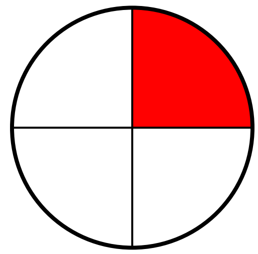
Let's look at each of the pizzas made for the three different families, and represent them using fractions.

Fractions are most often used to represent part of a whole. We would write this as

$$\frac{part}{whole}$$

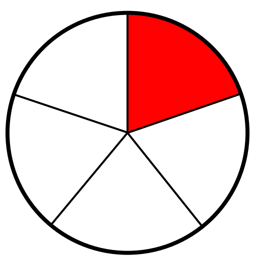
The family of four gets a pizza cut into 4 equal slices. In other words, the whole pizza is made of 4 slices. If we focus on one individual slice, we are looking at 1 part of the whole pizza. The drawing to the right can be represented by the following fraction.

$$\frac{1}{4}$$

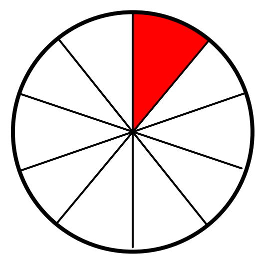


We know this because we are looking at 1 part of the 4 slices that make up the whole.

If we look at the other families' orders, we represent them as follows.



$$= \frac{1}{5}$$



$$= \frac{1}{10}$$

In each of the drawings, we are shading in 1 part of the whole pizza. The whole pizza is made up of the total number of slices. As you can see, the individual slices can be represented by fractions. In these fractions, the part is the **numerator**, and the whole is the **denominator**.

- The top number of a fraction is called the **numerator**.
- The bottom number is called the **denominator**.

$$\frac{\text{numerator}}{\text{denominator}}$$

The denominator tells you what kind of pieces we are talking about. For example, $\frac{1}{2}$ means we're talking about halves, $\frac{1}{3}$ means thirds, $\frac{1}{4}$ fourths, $\frac{1}{5}$ fifths, and so on. The numerator tells how many of those pieces – halves, thirds, fourths, fifths, etc. – you have.

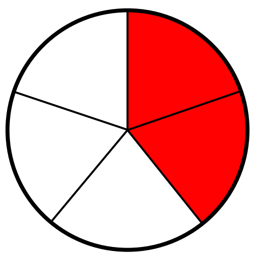
In the pizza problem, we focused on 1 individual slice, but what if we wanted to focus on multiple slices?

Example

Two people in the family of five decided to save their slices of pizza for later. What fraction of the whole pizza was saved for later?

Solution

In this case, there are 5 total slices, and 2 of them were saved. The denominator will be 5 because there are 5 total slices. The numerator will be 2 because we are focusing on 2 pieces from the total of 5 slices.

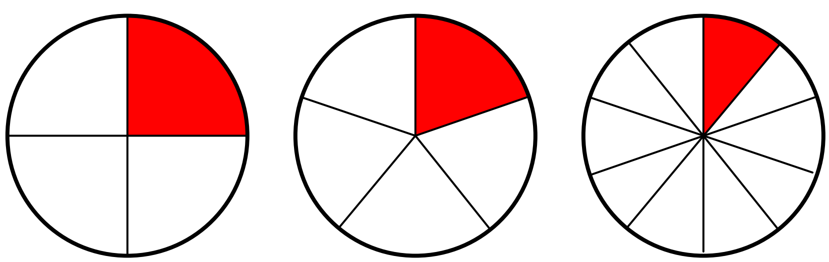


$$= \frac{2}{5}$$

So, $\frac{2}{5}$ of the pizza is saved for later.

You may wonder why we use pictures to represent these fractions. The pictures help us create a visual representation of the fractions, and they help us compare fractions.

Let's revisit the initial pizza problem.



As we can see, the shaded region is the largest in the left most circle and the smallest in the right most circle. If we were to represent these shaded regions using fractions we could say the following:

$$\frac{1}{4} > \frac{1}{5} > \frac{1}{10}$$

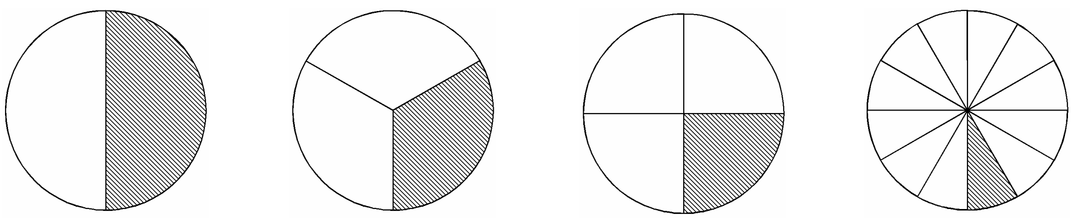
This method for comparing fractions only works if we use the same object to represent one whole. In this case, we used identical circles to represent one whole pizza.

Example

Compare the size of the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, and $\frac{1}{12}$, and order them from least to greatest.

Solution

We will use pictures to help us answer this question.



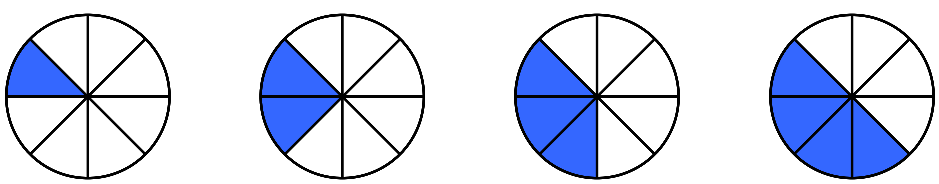
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{12}$

In the picture provided, we use the same size circle to represent one whole.

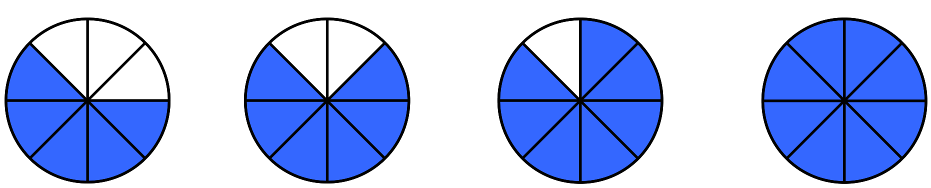
Just as with the pizza story, we see that when the same shape is divided into equally sized pieces, the more pieces there are, the smaller each piece is. So the answer is

$$\frac{1}{12}, \quad \frac{1}{4}, \quad \frac{1}{3}, \quad \frac{1}{2}$$

The rule is: if a set of fractions has the same numerator, the one with the <u>smallest</u> denominator is largest in value. What if fractions have the same denominators, but different numerators? Look at the example below. What do you notice?



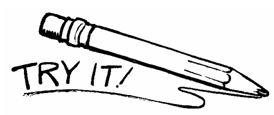
$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$



$\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

As the numerator grows, so does the value of the fraction. Why is this? As it says above, the numerator tells you "how many" of what you are considering. So, if we have the fraction, $\frac{1}{5}$, we know from the denominator that we are using fifths, and we only have one of these fifths. How does this compare to the fraction $\frac{2}{5}$? We're still using fifths, but now we have two of them; $\frac{2}{5}$ must be bigger than $\frac{1}{5}$.

Based on this information, try some of these problems on your own.



1. Circle which fraction is larger.

a) $\frac{1}{11}$ or $\frac{1}{9}$

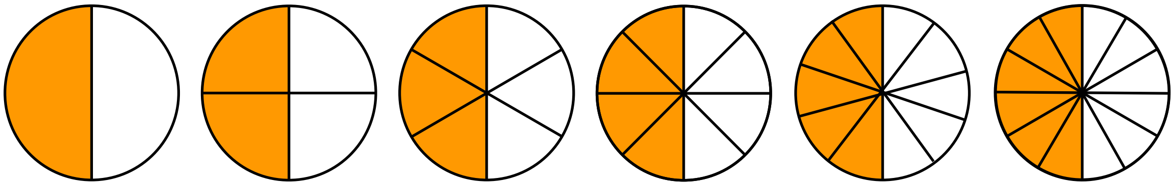
b) $\frac{6}{17}$ or $\frac{6}{15}$

c) $\frac{13}{19}$ or $\frac{11}{19}$

Sometimes it is helpful to compare unfamiliar fractions with more familiar numbers to see how big they are. For instance, think about $-\frac{2}{3}$, and $\frac{1}{3}$. At first you might be tempted to say that $-\frac{2}{3} > \frac{1}{3}$ since the numerator of $-\frac{2}{3}$ looks bigger than the numerator of $\frac{1}{3}$. But compared with 0, we know that negative numbers are always less than zero, and positive numbers are always greater than zero, so $-\frac{2}{3} < \frac{1}{3}$.

Another interesting fact about fractions is that, although they may look different, they may be equal in value.

Look at a picture form of these fractions,



$$\frac{1}{2} \qquad \frac{2}{4} \qquad \frac{3}{6} \qquad \frac{4}{8} \qquad \frac{5}{10} \qquad \frac{6}{12}$$

We can see that they are all equal because the same total amount is colored every time. The only difference is how many equal pieces each circle is divided into. No matter how many pieces each circle has, the total amount is still the same – one whole!

Let's look at the actual fractions and see why they are all equal, using algebra. Once again, the equal forms of $\frac{1}{2}$ are

$$\frac{1}{2}, \quad \frac{2}{4}, \quad \frac{3}{6}, \quad \frac{4}{8}, \quad \frac{5}{10}, \quad \frac{6}{12}, \quad \ldots$$

Do you see a pattern with the numbers? Let's rewrite the fractions to show more clearly how they are equal.

$$\frac{1}{2}=\frac{1\times 1}{2\times 1}=\frac{1}{2}\left(\frac{1}{1}\right)$$

$$\frac{2}{4}=\frac{1\times 2}{2\times 2}=\frac{1}{2}\left(\frac{2}{2}\right)$$

$$\frac{3}{6}=\frac{1\times 3}{2\times 3}=\frac{1}{2}\left(\frac{3}{3}\right)$$

$$\frac{4}{8}=\frac{1\times 4}{2\times 4}=\frac{1}{2}\left(\frac{4}{4}\right)$$

$$\frac{5}{10}=\frac{1\times 5}{2\times 5}=\frac{1}{2}\left(\frac{5}{5}\right)$$

$$\frac{6}{12}=\frac{1\times 6}{2\times 6}=\frac{1}{2}\left(\frac{6}{6}\right)$$

> **FACT**
>
> *Any number divided by itself equals one. For example*
>
> $\frac{5}{5}=1$ *and* $\frac{z}{z}=1$

Each equal form of $\frac{1}{2}$ is just $\frac{1}{2}$ multiplied by something equal to one!

> **FACT**
>
> *The __identity property of multiplication__ states that any number multiplied by 1 is that number. It allows us to multiply any fraction by 1 without changing its value!*

If each fraction above can be written as $\frac{1}{2}\times 1$, and the identity property says that $\frac{1}{2}\times 1=\frac{1}{2}$, then we have just proven that all of the different fractions equal $\frac{1}{2}$. Hooray!

We can use the identity property to find equivalent fractions for *any* fraction. Let's try an example.

Example

Write two fractions that are equivalent to $\frac{1}{3}$.

Solution

If we multiply both the numerator and denominator by 2, we see that

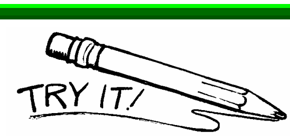
$$\frac{1}{3} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$$

Or we can multiply both the numerator and denominator by 3 and get

$$\frac{1}{3} = \frac{1\times 3}{3\times 3} = \frac{3}{9}$$

So, $\frac{2}{6}$ and $\frac{3}{9}$ are equivalent to $\frac{1}{3}$.

Again, we are able to do this because we are multiplying by something equal to 1, so the fractions are equivalent.



2. Write two fractions that are equivalent to:

a) $\frac{3}{5}$

b) $\frac{2}{3}$

3. Complete the equivalent fraction

a) $\frac{3}{5} = \frac{12}{\square}$

b) $\frac{16}{24} = \frac{\square}{12}$

Equivalent fractions are useful in rewriting fractions using smaller numbers.

For example, $\frac{30}{45}$ is a fraction with large numbers, but we can simplify it using equivalent fractions and common factors.

- A fraction is in **simplest form** or **lowest terms**, if the numerator and denominator share no common factors. Thus, the fraction has no equivalent forms with smaller numbers.

Simplest form and lowest terms mean the exact same thing, so don't get confused when you see either one. "Simplify" also means to put something into simplest form.

For instance, $\frac{6}{9}$ is not in lowest terms, because the numerator and denominator share a common factor of 3. $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

Algorithm

To put a fraction in lowest terms:

1. Find the Greatest Common Factor of the numerator and denominator.
2. Divide the numerator and denominator by that factor.

Remember, if the numerator and denominator of a fraction have no factors in common, then it is already in its simplest form. Additionally, if a fraction is in simplest form, the GCF of its numerator and denominator is 1.

Example

Write $\frac{30}{45}$ in lowest terms.

Solution

Steps 1 & 2: Write the factors of the numerator and denominator.

Underline, circle, or highlight the factors they have in common.

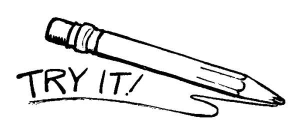
Factors of 30: **1**, 2, **3**, **5**, 6, 10, **15**, 30

Factors of 45: **1**, **3**, **5**, 9, **15**, 45

Step 3: We see that fifteen is the greatest common factor.

Step 4: Divide the numerator and denominator by the GCF.

$$\frac{30}{45} = \frac{30 \div 15}{45 \div 15} = \frac{2}{3}$$



4. Write the following fractions in simplest form.

a) $\frac{4}{12}$ b) $\frac{6}{15}$ c) $\frac{4}{5}$

One way to check if fractions are equivalent is the **cross product** method.

- The **cross product** shows that the product of the means equals the product of the extremes in equivalent fractions. We will explore the definition of the underlined terms in more detail when we discuss proportions in Lesson 16.
- The numerator of the first fraction multiplied by the denominator of the second fraction is equal to the product of the denominator from the first fraction and the numerator from the second fraction.

If $\frac{1}{2}=\frac{2}{4}$, then $(1\times 4)=(2\times 2)$. If $\frac{a}{b}=\frac{c}{d}$, then $(a\times d)=(b\times c)$.

We call it the cross product because we multiply across the equals sign as shown

Example

Show that $\frac{1}{4}=\frac{3}{12}$ using the cross product.

Solution

Write down the fraction and show the cross product.

$$\frac{1}{4}=\frac{3}{12}$$

$$(1\times 12)=(3\times 4)$$

$$12=12 \quad \checkmark$$

Since the cross products are equal, the fractions are equivalent.

Review

1. Highlight the following definitions:
    a. fraction
    b. numerator
    c. denominator
    d. simplest form
    e. lowest terms
    f. cross product

2. Highlight the "Algorithm" box.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

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Practice Problems

Math On the Move Lesson 5

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 5, Set A and Set B.

Set A

1. Which fraction is *not* equivalent to $\frac{2}{3}$? (Circle the correct answer)

    A. $\frac{2}{4}$ B. $\frac{6}{9}$ C. $\frac{4}{6}$ D. $\frac{20}{30}$

2. State whether the fraction is in simplest form. If it is not, then simplify it.

a) $\frac{8}{16}$ b) $\frac{12}{18}$ c) $\frac{9}{10}$ d) $\frac{13}{64}$

Set B

1. How many equivalent fractions are there for any fraction? Explain your reasoning.
2. Put the fractions in order from least to greatest (smallest to biggest). A number line may be helpful.

$$\frac{1}{2},\ \frac{3}{4},\ -\frac{1}{5},\ \frac{3}{5},\ -\frac{2}{3},\ -\frac{3}{5},\ 0,\ 1,\ -\frac{1}{2}$$

3. What does it mean when a fraction's numerator is <u>greater than</u> its denominator? How does it compare to the value 1? For example, is the value of $\frac{4}{3}$ smaller or bigger than 1? Explain your answer using pictures and/or words.

1. a) $\frac{1}{11}$ or $\frac{1}{9}$ b) $\frac{6}{17}$ or $\frac{6}{15}$ c) $\frac{13}{19}$ or $\frac{11}{19}$

2. a) $\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\ldots$ b) $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\ldots$

3. a) $\frac{3}{5}=\frac{12}{20}$ b) $\frac{16}{24}=\frac{8}{12}$

4. a) $\frac{4}{12}=\frac{1}{3}$ b) $\frac{6}{15}=\frac{2}{5}$ c) $\frac{4}{5}$ is in simplest form

End of Lesson 5