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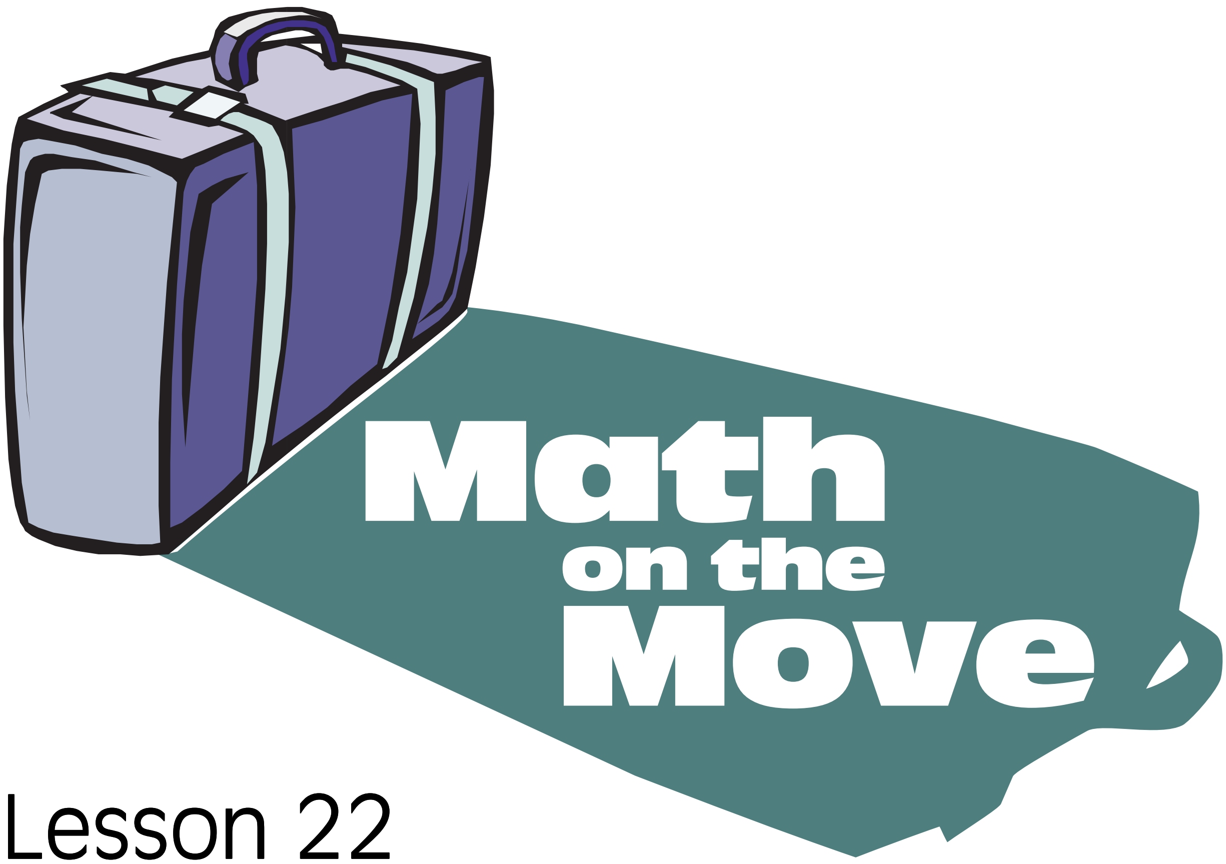

Lesson 22

3-Dimensional Solids

Objectives

- Classify 3-Dimensional solids
- Determine the Volume of 3-Dimensional solids

Authors:

Jason March, B.A.
Tim Wilson, B.A.

Editor:

Linda Shanks

Graphics:

Tim Wilson
Jason March
Eva McKendry

National PASS Center
BOCES Geneseo Migrant Center
27 Lackawanna Avenue
Mount Morris, NY 14510
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass

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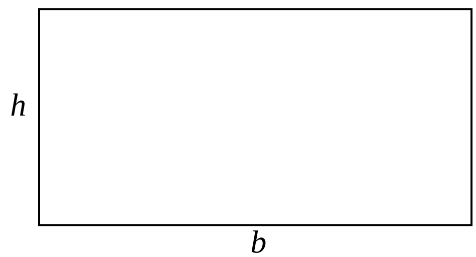
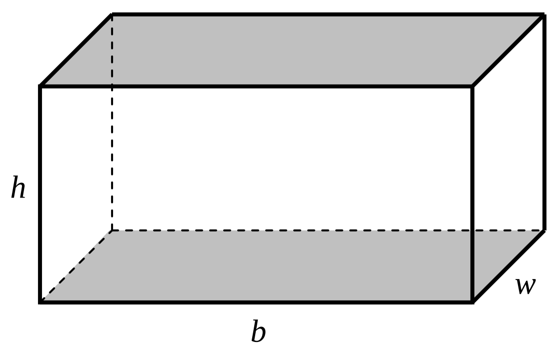
In the last several lessons, we have discussed different types of geometric figures. Our discussion has been limited to two-dimensional (2-D) objects. However, we live in a three-dimensional (3-D) world. Now let's discuss objects that exist in the three-dimensional world.

For example, a rectangle is a 2-D object. It only has two dimensions – a base and a height.



> **FACT**
>
> - *b (base) can also be represented by l (length).*
> - *In a 2-D figure, h (height) can be called w (width).*
> - *In a 3-D figure, w (width) might be called d (depth).*

But, what if we added a third dimension to the rectangle?



Here we have an object made of three dimensions: base (*b*), height (*h*), and width (*w*).

This shape is known as a **rectangular prism**.

- A **rectangular prism** is a three-dimensional solid. It has two parallel bases that are congruent rectangles. In the figure drawn, the <u>bases are shaded gray</u>.

This may be difficult to see on paper so let's think of some real life examples of this solid.

- A shoebox
- A textbook
- <u>A refrigerator</u>

> **FACT**
>
> *3-D objects are often called solids.*

The whole world is made of three-dimensional solids. There are more than just rectangular prisms. In fact, a **prism** can define many types of solids.

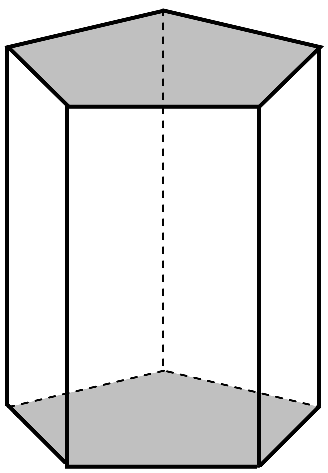
- A **prism** is a solid figure. It has two parallel bases that are <u>congruent polygons</u>.

A prism can have bases shaped like any polygon.

Example

Classify the following prisms.

a)



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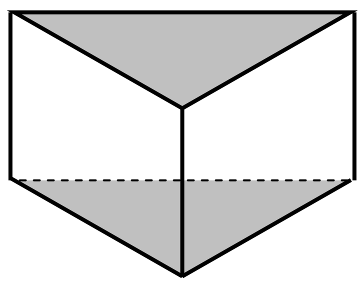

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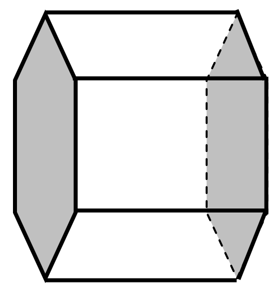


Solution

We know these are all prisms, so we need to locate the base and determine which polygon it is.

a) The bases on this prism are shaded in gray. Each bases has five vertices and sides, so they are pentagons. This means that the solid is a <u>pentagonal</u> prism.

b) The bases of this prism have three vertices and sides. The polygon with three sides is a triangle, so this is a <u>triangular</u> prism.

c) The bases on this prism have six vertices and sides. The six sided polygon is a hexagon, so this is a <u>hexagonal</u> prism.

Notice, when the base is a polygon with five or more sides, the prism name ends in "-al". We add "-al" to the name of the polygon.

- Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal, Decagonal

Another shape that has two parallel, congruent bases is a **cylinder**. However, a cylinder is not considered a prism.

- A **cylinder** is a solid with two parallel, congruent bases that are circles.
    - o The following is an example of a cylinder

Again, this may be hard to see on paper, so let's think of some objects that are shaped like a cylinder.

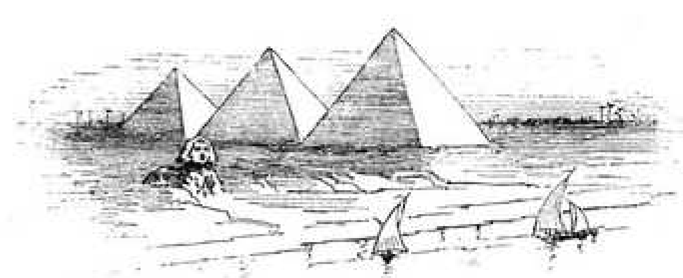
- o <u>A can of beans</u>
- o A marker
- o A pipe

Think Back

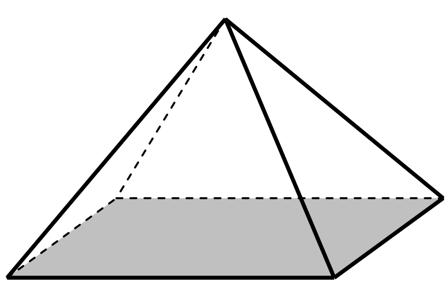
A cylinder is not a prism because its base is a circle, and a circle is not a polygon.

There are three dimensional shapes that have only one base. Think of the ancient Egyptians and the tombs they built for their Pharaohs. The tomb shape is more commonly known as a **pyramid**.

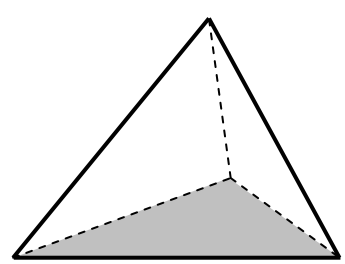
- A **pyramid** is a 3-D solid. Its base is a polygon, and its sides are triangles that converge to a common point.



The pyramids built by the ancient Egyptians are <u>square pyramids</u>. Their bases are squares.



Square Pyramid

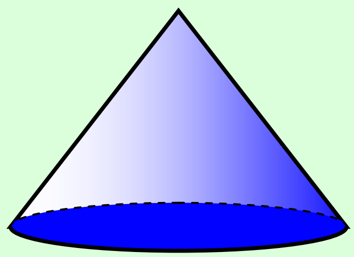


Triangular Pyramid

Most pyramids have a square or triangular base. Other kinds of bases are possible, as well.

A shape similar to a pyramid is a **cone**.

- A **cone** is a solid with one circular base and one vertex.
  - The following is an example of a cone.

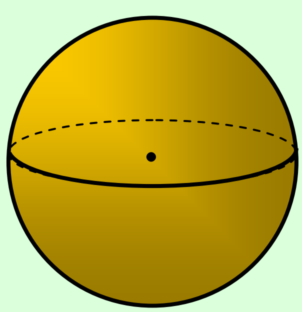


A cone is a familiar shape that we see all the time. Some examples of cones are:

- o A traffic cone
- o A megaphone
- o An ice cream cone

The last 3-D shape we need to discuss has no base. This shape is known as a **sphere**.

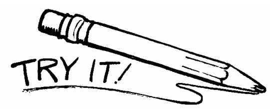
- o A **sphere** is a 3-Dimensional solid that is perfectly round. Every point on a sphere is the same distance from the center of the sphere.
  - The following is an example of a sphere.



The sphere is the hardest shape to see on paper. Some real life examples of a sphere include:

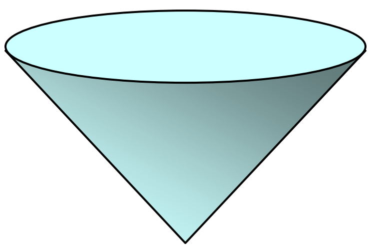
- A soccer ball
- A globe (of the Earth)
- A baseball
- An orange

Now we have discussed all the basic solids. Try classifying some on your own!



1. Classify the following solid figures.

a)



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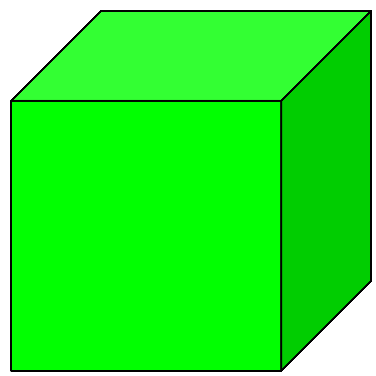

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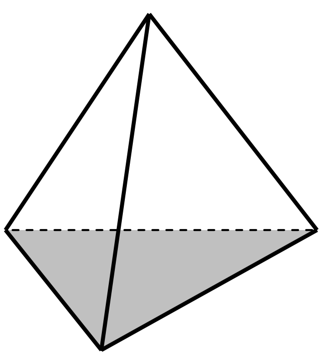

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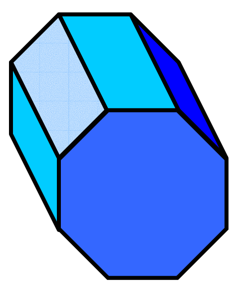
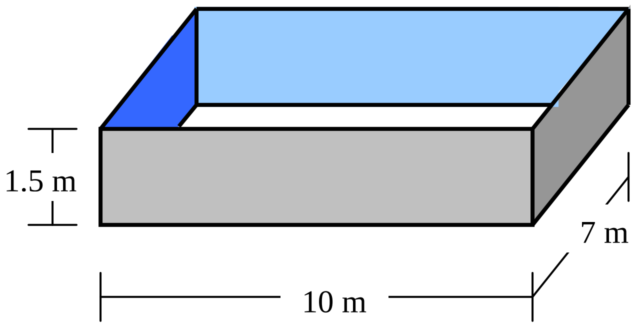


Now that we know about 3-D solids, we can solve problems using them.

Your friend, Daniela, just finished building a rectangular pool. She wants to know how much water she needs to fill it. She knows $1\text{ kL} = 1\text{ m}^3$, but she does not know how many cubic meters are in her pool.

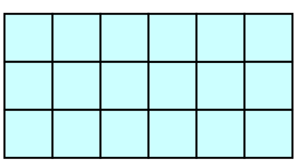


A cubic meter is a measure of volume. A kiloliter is a measure of capacity. Volume is the amount of space a solid takes up. Capacity is the amount that an object can hold. So if $1\text{ kL} = 1\text{ m}^3$, *then* 1 kL *takes up* 1 m^3 *of space. This is why Daniela needed* kL *of water, not* m^3 *of water, to fill her pool.*

Daniela shows you a picture of her pool. You can help her find out how much water she needs. Her pool is a rectangular prism. To find out how much water she needs, you need to find the **volume** of the rectangular prism.

- **Volume** is the amount of space that a solid takes up. Volume is measured in cubic units.

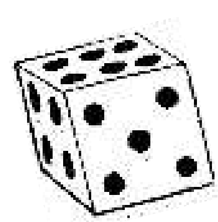
Volume is very similar to area. When you wanted to find the area of rectangles, you added up the square units.

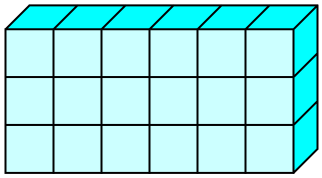


This rectangle has an area of 18 square units. Initially, we found this by counting up all the squares.

If we wanted to find the volume of a rectangular prism, we just add up all the **cubes**.

- A **cube** is a rectangular prism with 6 congruent square faces.
    - A standard six-sided die is an example of a cube

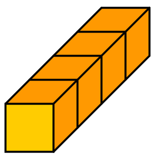


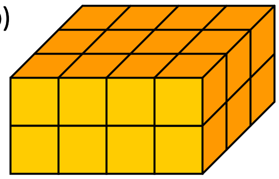


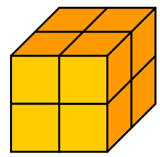
The volume of this rectangular prism is 18 <u>cubic units</u>, because there are 18 cubes in total.

Example

Find the volume of the following rectangular prisms

a) 

b) 

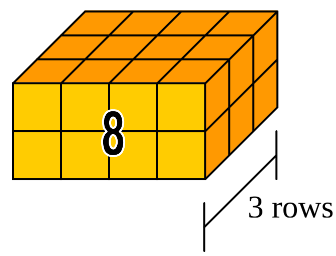
c) 

Solution

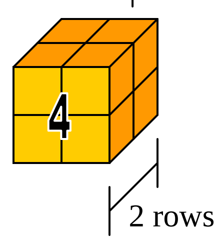
To find the volume of the given solids, we simply add up all the cubes.

a) We can see all the cubes in this rectangular prism. The dimensions of the prism are 1 by 1 by 4. There are 4 total cubes in the prism, so the volume is 4 cubic units, or 4 units^3.

b) In this prism, we cannot see all the cubes. We have to figure out how many cubes there are using reasoning. The front face of the prism has 8 cubes in it. We can see that there are 3 rows with 8 cubes in it. We can assume that there are $3 \times 8 = 24$ cubes. This means the volume is 24 cubic units, or 24 units^3.



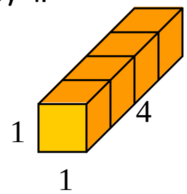
c) In this prism, we cannot see all the cubes. We can see that the front face of the prism has 4 cubes in it. The prism also has two rows with 4 cubes in it. We can assume there are $2 \times 4 = 8$ cubes. This means the volume is 8 cubic units, or 8 units^3.



Remember that with rectangles, we can multiply the dimensions to find the area. What do you notice about the dimensions of the rectangular prism and the volume?

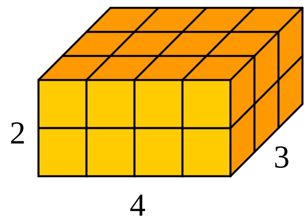
In example “a,” the dimensions are 1 by 1 by 4.

The volume is $1 \times 1 \times 4 = 4$ cubic units.



In example “b,” the dimensions are 2 by 4 by 3.

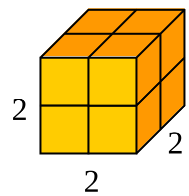
The volume is
$2 \times 4 \times 3 = 24$ cubic units.



> ***Think Back***
>
> *Remember that with rectangles, dimensions were said base by height. With rectangular prisms, they are said length by width by height.*

In example “c,” the dimensions are 2 by 2 by 2.

The volume is $2 \times 2 \times 2 = 8$ cubic units.



The examples show us that volume is found by multiplying the three dimensions of the prism.

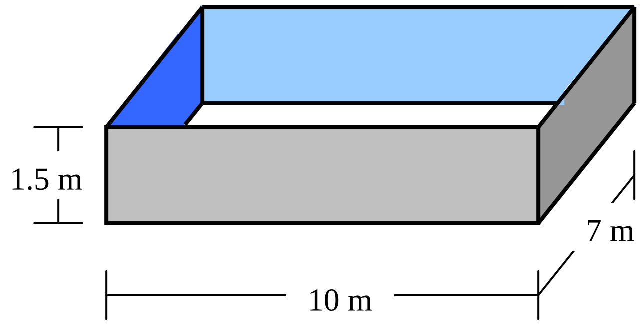
The volume formula for a rectangular prism is

$$V = l \times w \times h$$

Volume = length × width × height

> **FACT** *The three dimensions can be called a variety of names. We use length, width, and height as a standard for the formula.*

Let's look back at the problem with Daniela's pool. She gave us the dimensions of the pool.



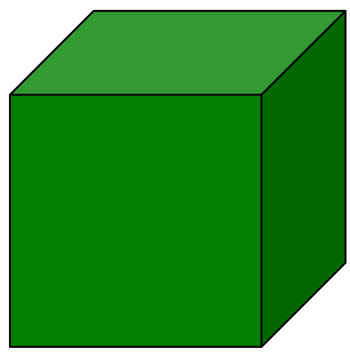
So, to find the volume of the pool, we simply multiply all three dimensions.

$$V = 1.5 \times 10 \times 7 = 105 \text{ m}^3$$

If $1 \text{ kL} = 1 \text{ m}^3$, Daniela needs 105 kL of water to fill the pool.

Example

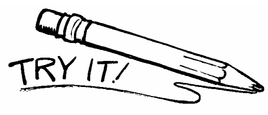
Find the volume of the following cube.



Solution

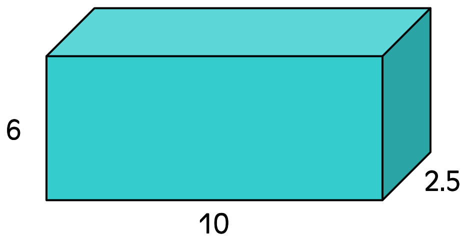
To find the volume of the cube, we need to know its three dimensions. Since a cube is made of congruent squares, the lengths of all the edges are the same. So, the three dimensions of this cube are 3 by 3 by 3. The volume is $3 \times 3 \times 3 = 27 \text{ in.}^3$

Notice that we multiplied the edge by itself three times. Remember that repeated multiplication is the same as using an exponent. So, the volume formula for a cube is $(\text{the length of one edge})^3$. *The term "cubed" comes from the volume of a cube.*



2. Find the volume of the following rectangular prisms.

a)



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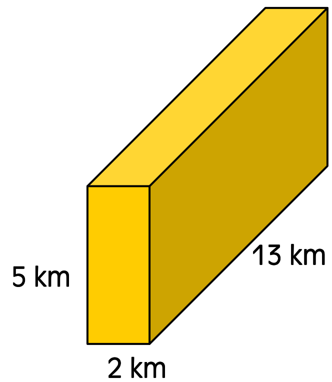

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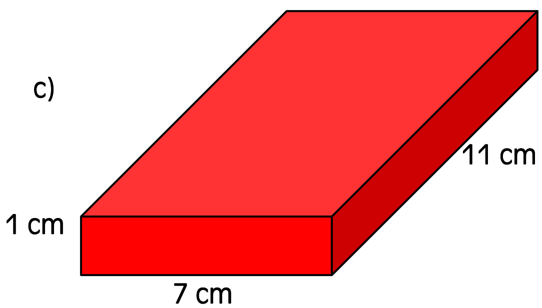

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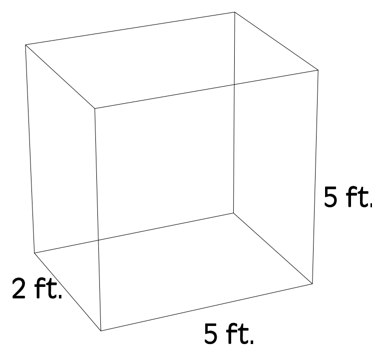


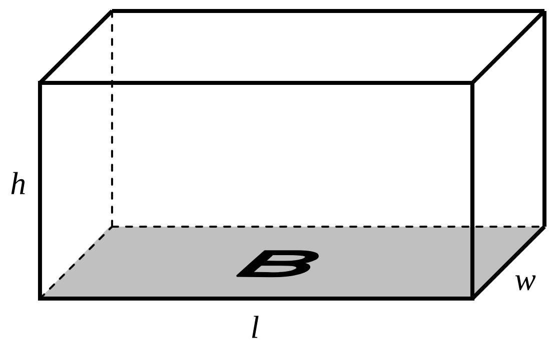
The volume of a rectangular prism is found by multiplying all three dimensions. However, there is another formula that we use for all prisms.

$$V = B \times h$$

Where <u>*B* is the area of the base of the prism</u> and *h* is the height. When we were finding the volume of rectangular prisms, we simply broke down the area of the base into the length and width.

Think Back

$A = bh$ is not the same as $V = Bh$. We use the lowercase "b" when we are describing the length of the base. We use the uppercase "B" when we are describing the area of the base.



The area of the base of this rectangular prism is $l \times w$. This means the volume of the rectangular prism is.

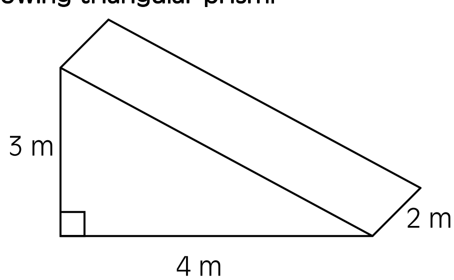
$$V = B \times h$$

$$= l \times w \times h$$

Now that we know this, we can use the new formula to find the volume of triangular prisms.

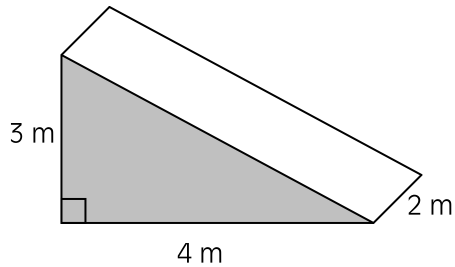
Example

Find the volume of the following triangular prism.



Solution

First, we must find the base of the prism. Since it is a triangular prism, the base is the right triangle on the front side of the prism.



Next, we need to find the area of that triangle in front.

$$B = \frac{1}{2}bh$$
$$= \frac{1}{2}(4)(3)$$
$$B = 6 \text{ m}^2$$

> ***Think Back***
>
> *Area formula for a triangle is* $A = \frac{1}{2}bh$.

Now that we have the area of the base, we can substitute, or plug that into the formula to find the volume of the prism.

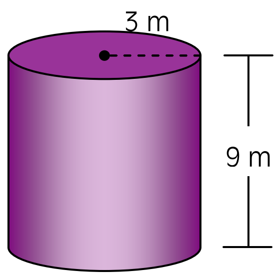
$$V = Bh$$
$$= 6(2)$$
$$V = 12 \text{ m}^3$$

Be careful when you are plugging in values for the formula. We used 2 as the height of the prism, because we already used 3 and 4 to find the area of the base, *B*.

Even though the cylinder is not a prism, we can use the same formula for finding the volume. This works because, just like a prism, a cylinder has two congruent bases.

Example

Find the volume of the following cylinder. (Round your answer to the nearest hundredth)



Solution

To find the volume of the cylinder, we must use the volume formula.

$$V = Bh$$

A cylinder has two circular bases, so to find the area of the base, *B*, we need to find the area of the circle. The circle has a radius of 3 meters. We plug that value into the area formula for a circle.

$$\begin{aligned} B &= \pi r^2 \\ &= \pi(3)^2 \\ B &= 9\pi \end{aligned}$$

> ***Think Back***
>
> *Area formula for a circle is $A = \pi r^2$.*

We will use 3.14 to estimate pi.

$$\begin{aligned} B &= 9\pi \\ &\approx 9(3.14) \\ B &\approx 28.26 \text{ m}^2 \end{aligned}$$

Now that we have found the area of the base, *B*, we can plug that into the volume formula.

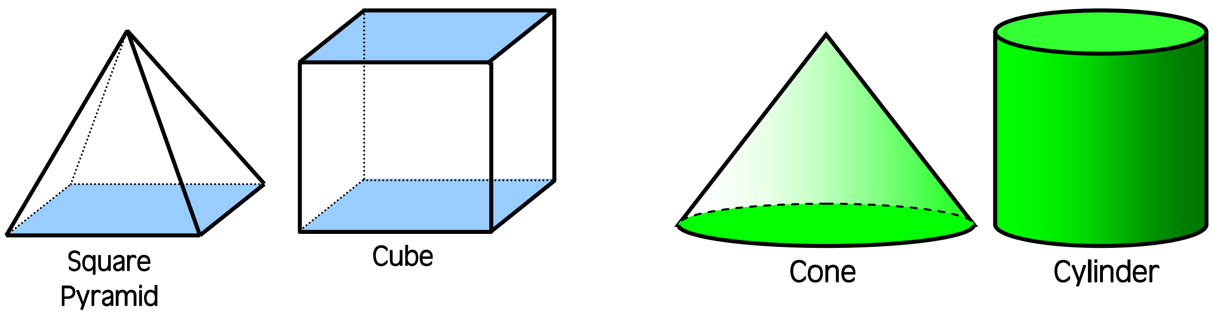
$$\begin{aligned} V &= Bh \\ &\approx (28.26)(9) \\ V &\approx 254.34 \text{ m}^3 \end{aligned}$$

So, the volume of the cylinder is approximately 254.34 m^3.

Remember, because 3.14 is an approximate value of pi, any answer involving it is approximate.

> **FACT**
>
> *When we are finding the length of an object, the answer is in units (ft., in., m, km, etc.).*
>
> *When we are finding the area of an object, the answer is in square units ($ft.^2, in.^2, m^2, km^2$).*
>
> *When we are finding the volume of an object, the answer is in cubic units ($ft.^3, in.^3, m^3, km^3$).*

Lastly, we need to determine the volume of the solids with one base. When we found the area of 2-D triangles, we compared them to parallelograms and found their area to be $\frac{1}{2}$ of a parallelogram. We will use a similar method for finding the volume of 3-D pyramids and cones. A square pyramid is similar to a cube because they both have a square base. However, the pyramid only has one base, while the cube has two.



Because we are working with 3-D solids, the volume of the pyramid is $\frac{1}{3}$ the volume of the cube. The volume of the cone is $\frac{1}{3}$ the volume of the cylinder. The pyramid and the cube, as well as the cone and the cylinder, share the following dimensions: the base, *B*, and the height, *h*.

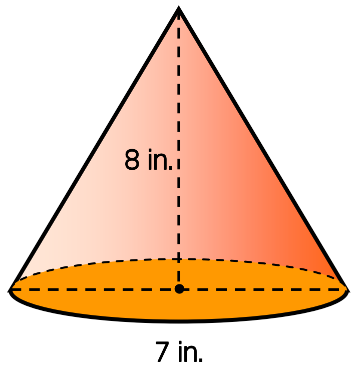
In a pyramid and a cone, the height is the perpendicular line from the base to the vertex where all the edges meet, as shown above. So, the volume formula for a pyramid (and a cone) is

$$V = \frac{1}{3}Bh$$

Where *B* represents the area of the base, and *h* represents the height.

Example

Find the volume of the following cone. (Round your answer to the nearest tenth)



Solution

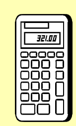
To find the volume of the cone, we must use the formula $V = \frac{1}{3}Bh$. Remember that *B* represents the area of the circular base. First, let's find *B*. We are given the diameter of the circular base, but the area formula for the circle requires the radius. The radius is half of the diameter, so the radius of the circle is $7 \times \frac{1}{2} = 3.5$. Now we can use that for the area of the base, *B*.

$$\begin{aligned} B &= \pi r^2 \\ &= \pi(3.5)^2 \\ &= 12.25\pi \end{aligned}$$

We will use 3.14 to estimate pi.

$$\begin{aligned} B &= 12.25\pi \\ &\approx 12.25(3.14) \\ B &\approx 38.465 \text{ in.}^2 \end{aligned}$$

Calculator Tip



To square difficult numbers, we use a calculator. Enter the number on the calculator and hit the square button.

x^2

Now that we know the area of the base, we can plug that into the volume formula.

$$V = \frac{1}{3}Bh$$
$$\approx \frac{1}{3}(38.465)(8)$$
$$\approx 102.57333 \text{ in.}^3$$

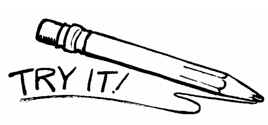
The problem asked us to round to the nearest tenth, so the volume of the cone is

$$V \approx 102.6 \text{ in.}^3$$

Think Back

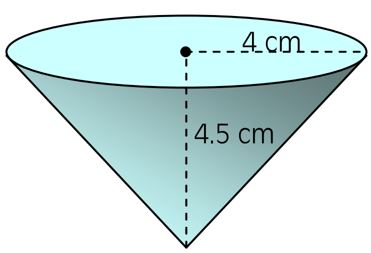
To round a number, look at the number to the right of the place value we are asked to round to. Then compare that number to 5.

Try to solve the following area problems on your own.

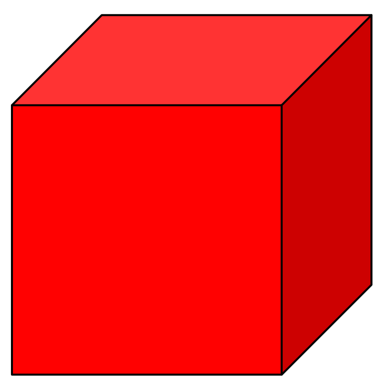


3. Find the volume of the following solids.
(Round to the nearest whole number)

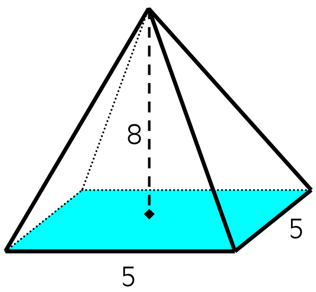
a) Cone



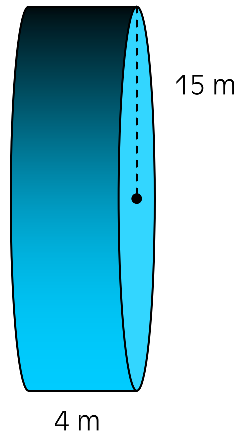
b) Cube



c) Square Pyramid



d) Cylinder



Review

1. Highlight the following definitions:
   a. rectangular prism
   b. prism
   c. cylinder
   d. pyramid
   e. cone
   f. sphere
   g. volume
   h. cube

2. Highlight all the volume formulas in the lesson.

3. Highlight all the "Fact" boxes.

4. Highlight all the "Think back" boxes.

5. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

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Practice Problems

Math On the Move Lesson 22

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 22, Set A and Set B.

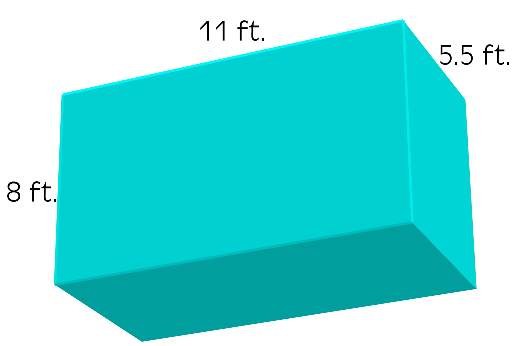
Set A

1. Which two types of solids have two bases?

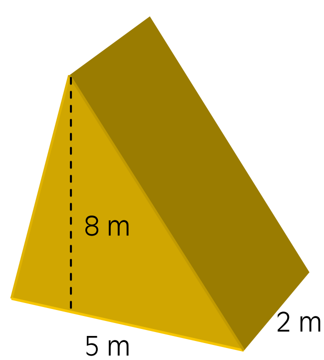
2. What do you know about the dimensions of a cube?

3. Find the volume of the following solids.

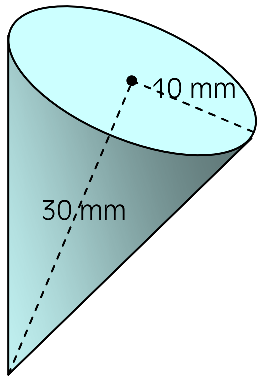
a)



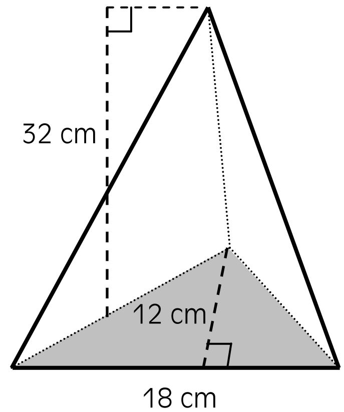
b)



c)



d)



Set B

1. Draw the solid described.

   a) A cylinder b) A pyramid with a square base c) A cone

2. What is the only solid that has no base?

3. If a cube has a volume of 64, what is the length of one edge?

ANSWERS TO TRY IT

1. a) Cone b) Rectangular prism (Cube) c) Triangular pyramid d) Octagonal prism

2. a) 150 units^3 b) 130 km^3 c) 77 cm^3 d) 50 ft.^3

3. a)
$$B = \pi(4^2)$$
$$= \pi(16)$$
$$\approx (3.14)(16)$$
$$B \approx 50.24 \text{ cm}^2$$
$$V \approx \frac{1}{3}(50.24)(4.5)$$
$$V \approx 75.36$$
$$V \approx 75 \text{ cm}^3$$

   b) $V = 2^3$ $V = 8 \text{ km}^3$

c) $B = 5 \times 5$

$B = 25 \text{ units}^2$

$V = \frac{1}{3}(25)(8)$

$V = 66.\overline{6}$

$V \approx 67 \text{ units}^3$

d) $B = \pi(15^2)$

$= \pi(225)$

$\approx (3.14)(225)$

$B \approx 706.5 \text{ m}^2$

$V \approx (706.5)(4)$

$V \approx 2826 \text{ m}^3$

End of Lesson 22