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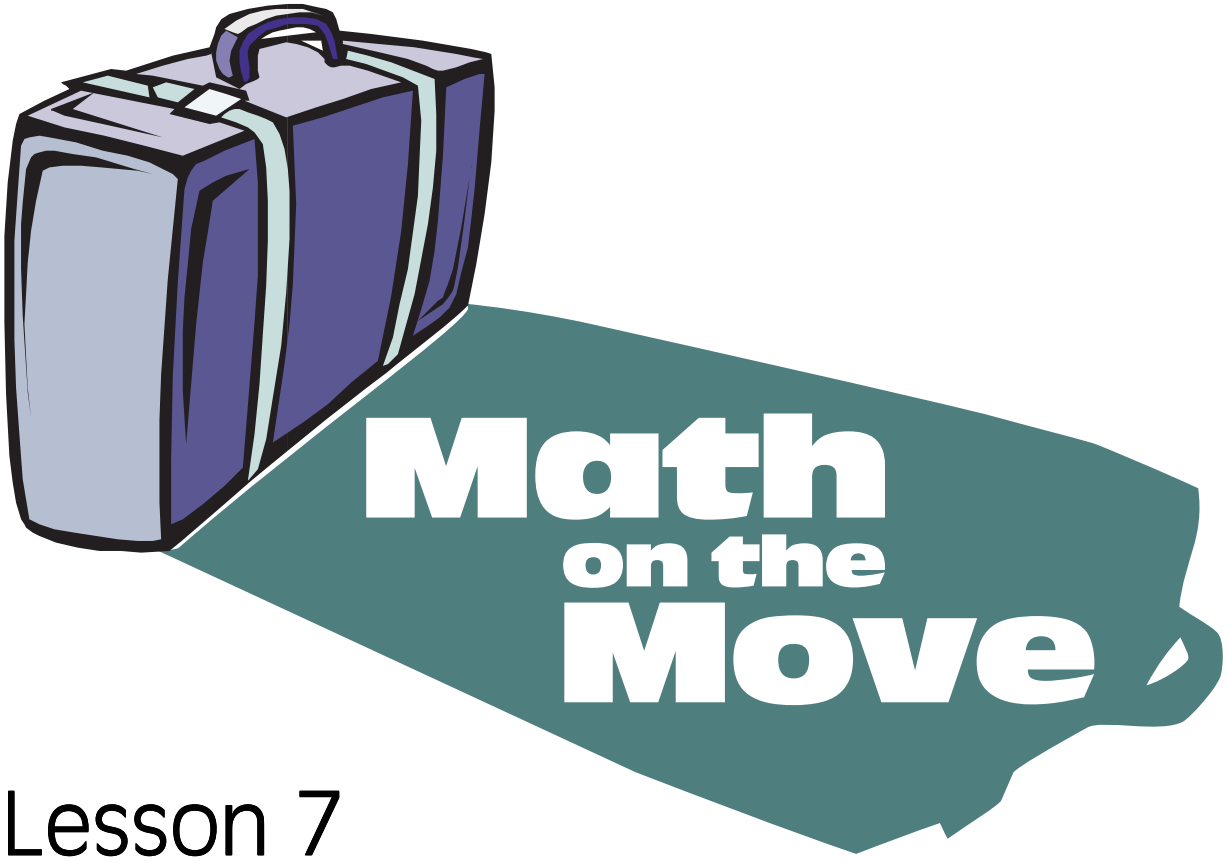

Lesson 7

Mixed Numbers

Objectives

- Understand the relationship between fractions and mixed numbers
- Convert between mixed numbers and improper fractions
- Add, subtract, multiply, and divide with mixed numbers

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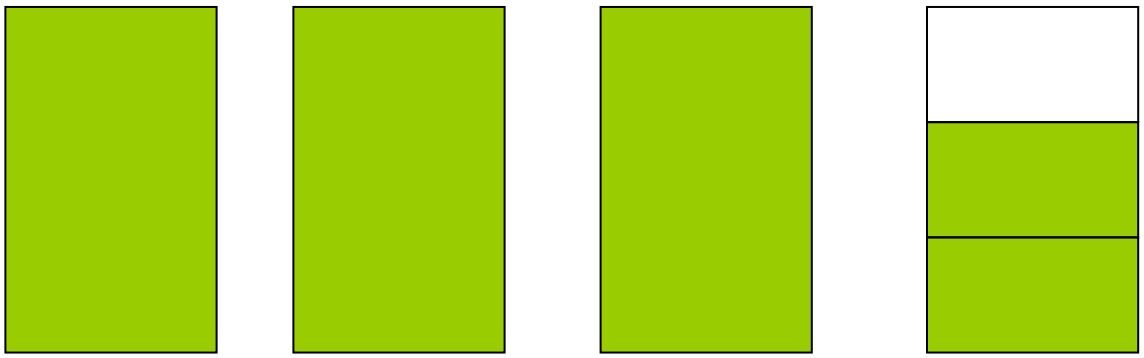
You are now very happy with your job as a grape picker. Every day, you are able to pick more and more grapes. One day, you were able to pick enough grapes to fill three large barrels and $\frac{2}{3}$ of a fourth barrel. You think to yourself, "I wonder how I can write this with numbers."

Now we are working with whole numbers and fractions. Let's look at a model of the grapes you picked.



$$1 + 1 + 1 + \frac{2}{3}$$

We know that $1+1+1+\frac{2}{3}=3+\frac{2}{3}$. In math, we see whole numbers and fractions being added very often. These numbers are called **mixed numbers**, and there is a special way to write them.

- A **mixed number** is the sum of a whole number, for instance 3, and a fraction, like $\frac{2}{3}$.

  When written, the addition sign is still there, but it is hiding.

  For instance, $3+\frac{2}{3} = 3\frac{2}{3}$, and $A+\frac{b}{c} = A\frac{b}{c}$.

A mixed number, such as $5\frac{3}{4}$ is said as, "five and three fourths."

It is called a "mixed" number because we are mixing whole numbers with fractions. Mixed numbers show how many wholes there are in a number that is not actually a whole number. Think back to the grapes. We didn't quite fill up the fourth barrel, but we have more than three barrels. $3\frac{2}{3}$ is between the whole numbers 3 and 4. When speaking, we say $3\frac{2}{3}$ as "three and two-thirds."

Mixed numbers can be made into fractions, too.

From the definition above, $3\frac{2}{3} = 3 + \frac{2}{3}$

Now we break up the 3, $= 1 + 1 + 1 + \frac{2}{3}$

Since each 1 is worth three thirds, this gives, $= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$

Now that we have common denominators, we can add our numerators. $= \frac{9}{3} + \frac{2}{3}$

$= \frac{11}{3}$

- A fraction with a numerator that is less than its denominator is called a **proper fraction**. A fraction with a numerator that is greater than or equal to its denominator, is called an **improper fraction**.

For instance $\frac{13}{3}$ is an improper fraction, because the top, 13, is bigger than the bottom, 3.

The fraction $\frac{7}{18}$ is a proper fraction, since 7 is smaller than 18.

Mixed numbers and whole numbers can always be shown as improper fractions.

When we made $3\frac{2}{3}$ into an improper fraction, we first noticed that $3\frac{2}{3}$ really meant "three wholes and two thirds." Then, we said that three wholes was the same as nine thirds. Last, we combined nine thirds with two thirds to get our answer, $3\frac{2}{3} = \frac{11}{3}$. This method can be used with any mixed number!

To convert a mixed number to an improper fraction :


Algorithm

1. Multiply the whole number part of the mixed number by the denominator of the fraction.
2. Add this product to the numerator of the fraction.
3. Put the fraction into simplest form.

$$5\frac{3}{4} = \overset{+}{5}\frac{3}{\underset{\times}{4}} = \frac{(5\times 4)+3}{4} = \frac{23}{4}$$

1. Write the mixed numbers as improper fractions.

a) $1\frac{1}{3}$ b) $2\frac{7}{8}$ c) $3\frac{3}{4}$ d) $5\frac{3}{5}$

What if we have an improper fraction and we want to make it into a mixed number?

Example

Write $\frac{9}{4}$ as a mixed number.

Solution

In this example, we are working with fourths. We already know that there are four fourths in every whole, because $\frac{4}{4}=1.$ Also, when looking at a fraction, remember that the numerator tells you how many halves, thirds, fourths, fifths, etc., that you have. In this case we have nine fourths. What we need to know is how many groups of four are in nine. If we divide nine by four, we see

$$\begin{array}{r} 2\ \text{R1} \\ 4\overline{)9} \\ \underline{8} \\ 1 \end{array}$$

There are two wholes and one remaining fourth, so our answer is,

$$\frac{9}{4} = 2\frac{1}{4}.$$

Think Back

Fractions mean division.

$$\frac{9}{4} = 9 \div 4$$

We can use this method to convert any improper fraction to a mixed number.

Algorithm

To convert an improper fraction to a mixed number :

1. Divide the fraction's numerator by its denominator.
2. The number of times it divides evenly is the "whole" part of the mixed number.
3. To the right of that, write a fraction. The numerator will be the remainder found in step 2, and the denominator will stay the same as that of the original fraction.

$$\frac{9}{2}$$

$$= 9 \div 2$$

$$= 4 \quad R \quad 1$$

$$= 4\frac{1}{2}$$

Example

Write $\frac{14}{3}$ as a mixed number

Solution

First write $\frac{14}{3}$ as a division problem.

$$3\overline{)14}$$

Find the quotient with the remainder.

$$\begin{array}{r} 4\ R\ 2 \\ 3\overline{)14} \\ \underline{-12} \\ 2 \end{array}$$

The 4 stays on the left, the 2 is the numerator,

and the 3 is the denominator.

$$4\ \frac{2}{3}$$

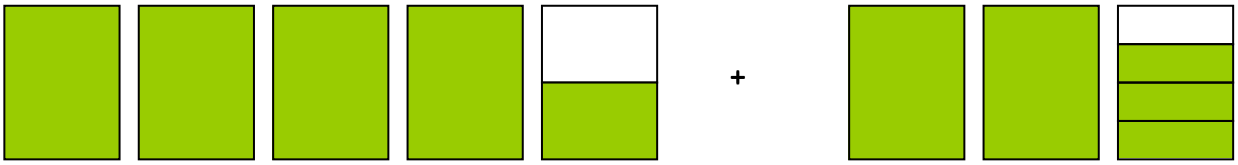
2. Write the improper fractions as mixed numbers in simplest form.

a) $\frac{5}{3}$ b) $\frac{21}{8}$ c) $\frac{5}{4}$ d) $\frac{11}{5}$

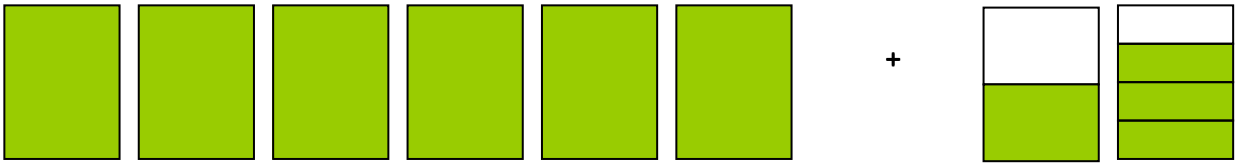
Notice that when $\frac{9}{4}$ is written as $2\frac{1}{4}$, it is much easier to tell how big it is just by looking at it.

As you continue to pick grapes, you begin to make friends. One of your friends, Kunal, wants to know how many grapes both of you picked together in one day. You picked $4\frac{1}{2}$ barrels of grapes, and he picked $2\frac{3}{4}$ barrels. How many total barrels of grapes did you and Kunal pick together?

We are trying to find $4\frac{1}{2} + 2\frac{3}{4}$. Visually, this is,



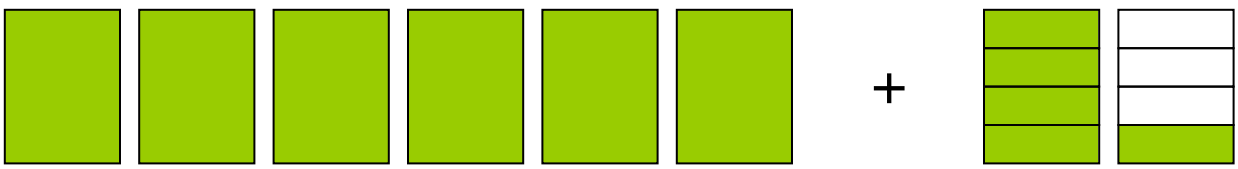
By looking at this, we can see that there are six full barrels and $\frac{1}{2}+\frac{3}{4}$ barrels left over.



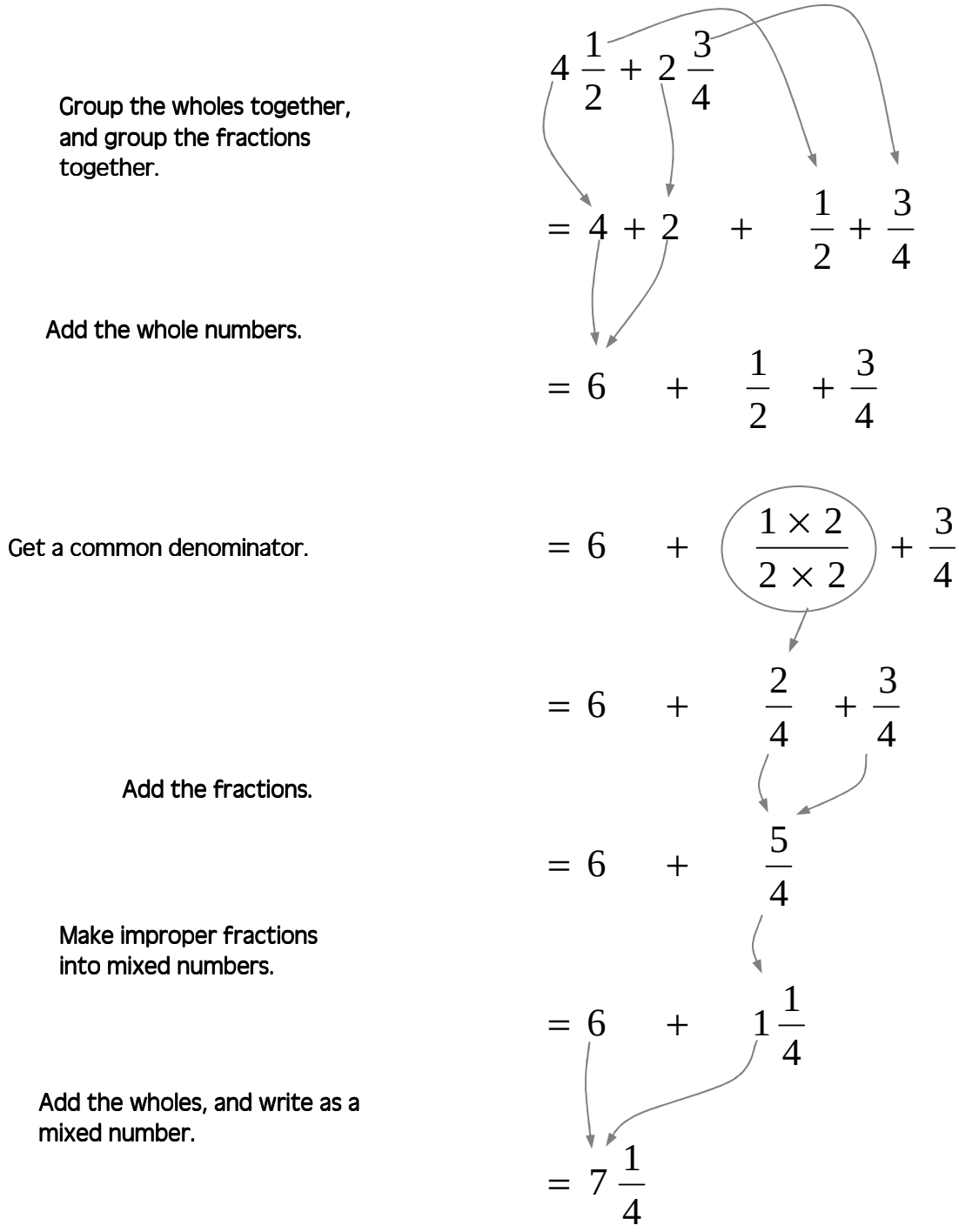
Remember, to find $\frac{1}{2} + \frac{3}{4}$, we must first find a common denominator. The lowest common denominator in this case is 4. So to change the denominator of 2 to a 4, we write,

$$\frac{1\times 2}{2\times 2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$

So we have $6 + \frac{5}{4}$. By looking at it, we can see that this is clearly $7\frac{1}{4}$.



This is how it works if we use only numbers.



Algorithm

To add mixed numbers:

1. Add the wholes.
2. Add the fractions.
3. If the sum of the fractions is improper, change the fraction to a mixed number, and add the wholes again.
4. Put the fraction in simplest form.

Here is a more challenging one.

Example

Simplify $2\frac{5}{9} + 4\frac{7}{8}$.

Solution

$$2\frac{5}{9} + 4\frac{7}{8}$$

$$= 2 + 4 + \frac{5}{9} + \frac{7}{8}$$

$$= 6 + \frac{5}{9} + \frac{7}{8}$$

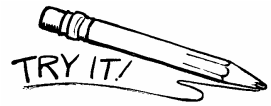
$$= 6 + \frac{5 \times 8}{9 \times 8} + \frac{7 \times 9}{8 \times 9}$$

$$= 6 + \frac{40}{72} + \frac{63}{72}$$

$$= 6 + \frac{103}{72}$$

$$= 6 + 1\frac{31}{72}$$

$$= 7\frac{31}{72}$$



3. Find the sums.

a) $1\frac{1}{3} + 2\frac{1}{3}$

b) $2\frac{1}{3} + 9\frac{5}{12}$

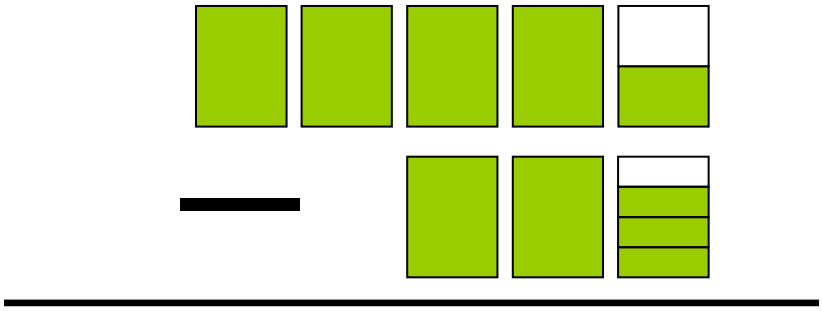
c) $4\frac{2}{5} + 3\frac{2}{3}$

After you tell Kunal how much you both picked, he wants to know how many more barrels of grapes you picked than he did. We can show him this too.

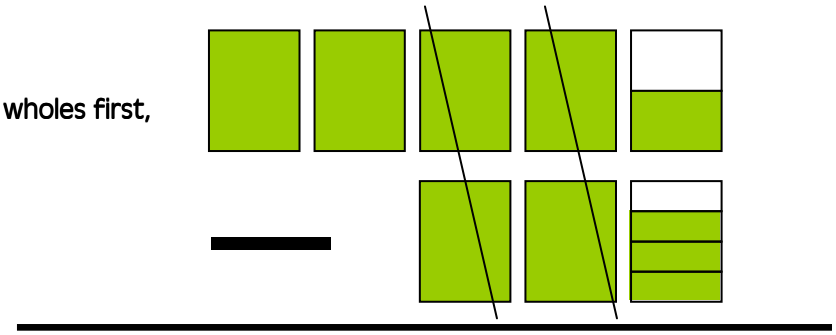
Remember that you picked $4\frac{1}{2}$ barrels of grapes, and he picked $2\frac{3}{4}$ barrels of grapes. We want to find how many more you picked, so we should find,

$$4\frac{1}{2} - 2\frac{3}{4}$$

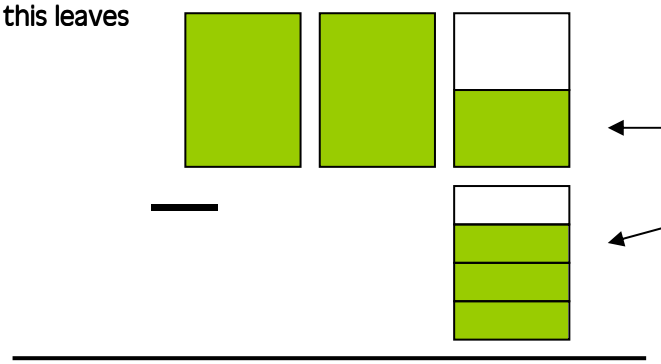
Using a model, this is



Take away the wholes first,



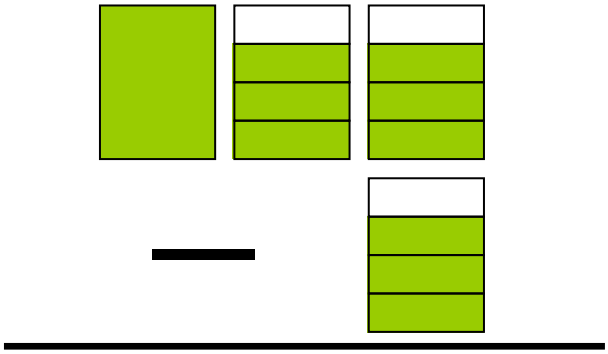
this leaves



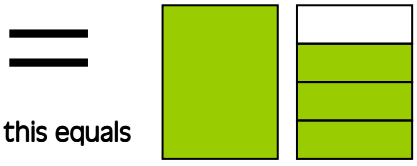
We cannot take $\frac{3}{4}$ of a barrel

from $\frac{1}{2}$ of a barrel

We will borrow one fourth from one of the whole barrels and subtract.



this equals



With numbers,

Subtract the whole numbers.

$$4\frac{1}{2} - 2\frac{3}{4}$$

$$= 2\frac{1}{2} - \frac{3}{4}$$

Get a common denominator.

$$= 2\frac{2}{4} - \frac{3}{4}$$

Since 2 is smaller than 3, we must borrow one whole, convert it to four fourths, and add it to the fraction.

$$= 1\frac{6}{4} - \frac{3}{4}$$

Subtract the fractions, and write the answer as a mixed number.

$$= 1\frac{3}{4}$$

So, you picked $1\frac{3}{4}$ more barrels of grapes than Kunal did.

Another way to do this problem is to convert everything to improper fractions.

$$4\frac{1}{2} - 2\frac{3}{4}$$

Bottom × Side + Top

$$= \frac{(2 \times 4) + 1}{2} - \frac{(4 \times 2) + 3}{4}$$

Simplify the numerators.

$$= \frac{9}{2} - \frac{11}{4}$$

Get a common denominator.

$$= \frac{9 \times 2}{2 \times 2} - \frac{11}{4}$$

$$= \frac{18}{4} - \frac{11}{4}$$

Subtract the fractions.

$$= \frac{7}{4}$$

Write as a mixed number.

$$= 1\frac{3}{4}$$

By now you probably see that borrowing can be tricky, and that sometimes, converting improper fractions is more difficult. This is why we have two separate methods for subtracting mixed numbers.


Algorithm

To subtract mixed numbers :

Method 1: The Borrowing Method

1. Leave in mixed number form.
2. Subtract the whole numbers.
3. Rewrite fractions with common denominators.
4. Borrow one whole if you need to.
5. Subtract the fractions.
6. Write in simplest form.

Method 2: Improper Fractions Method

1. Convert everything to improper fraction form.
2. Get a common denominator.
3. Subtract.
4. Write in simplest form (mixed number form).

Usually, if you are given very large numbers to subtract (for instance $107\frac{2}{3} - 103\frac{7}{12}$), the borrowing method makes the most sense to use. But when you are using relatively small numbers (such as $1\frac{1}{3} - \frac{2}{3}$), converting everything into improper fractions lets you avoid borrowing.

4. Subtract the following. Use whichever method you think is best.

a) $3\frac{3}{4} - 2\frac{1}{4}$

b) $4\frac{1}{3} - 3\frac{4}{5}$

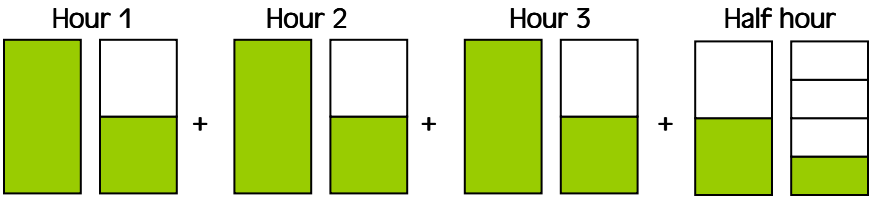
c) $16\frac{1}{5} - 14\frac{9}{15}$

Eventually, you get so fast at picking grapes that you want to time yourself doing it. You find that in one hour, you are able to pick $1\frac{1}{2}$ barrels of grapes. At that rate, how many barrels can you fill before lunch, which is $3\frac{1}{2}$ hours away?

If you pick $1\frac{1}{2}$ barrels in 1 hour, then in $3\frac{1}{2}$ hours, you must pick

Hours $\longrightarrow$ $3\frac{1}{2} \times 1\frac{1}{2}$ $\longleftarrow$ Barrels in one hour

Since we know multiplication is repeated addition, a picture of the grapes you pick in this time would look like



$$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$

Using what we know about addition,

$$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$

$$= 3 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$

$$= 5\frac{1}{4}$$

So you know that you can fill up $5\frac{1}{4}$ barrels before lunch.

But how do we show $3\frac{1}{2} \times 1\frac{1}{2}$ without repeated addition? We will use improper fractions.

$$3\frac{1}{2} \times 1\frac{1}{2}$$

Change to improper fractions.

$$= \frac{7}{2} \times \frac{3}{2}$$

Multiply numerator $\times$ numerator

and denominator $\times$ denominator

$$= \frac{21}{4}$$

Change back to a mixed number.

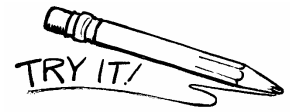
$$= 5\frac{1}{4}$$

<u>You should always use this method.</u>

Algorithm

To multiply mixed numbers:

1. Convert everything to improper fractions.
2. Multiply the numerators and denominators.
3. Write as a mixed number in simplest form.



5. Find the products in simplest mixed number form.

a) $1\frac{1}{3} \times 1\frac{1}{4}$

b) $2\frac{3}{4} \times 6\frac{4}{7}$

c) $\frac{9}{2} \times 3\frac{3}{5}$

After lunch, your boss tells you that you have to move all the grapes you picked that day into new barrels. The new barrels are really big. One of them can hold the amount of grapes that would normally fill $2\frac{1}{3}$ of the old barrels. How many of the new barrels are you going to need?

Out of the $5\frac{1}{4}$ barrels you picked before lunch, you need to know how many groups of $2\frac{1}{3}$ you have. Now we're talking about division! To answer the question, you need to find $5\frac{1}{4} \div 2\frac{1}{3}$. You know how to convert mixed numbers to improper fractions without much trouble, and you also know how to divide fractions by multiplying by the divisor's reciprocal. Let's use these steps to find the answer.

Change everything to improper fractions. $5\frac{1}{4} \div 2\frac{1}{3}$

$= \frac{21}{4} \div \frac{7}{3}$

Flip the divisor, and change the division sign to a multiplication sign.

$= \frac{21}{4} \times \frac{3}{7}$

Multiply numerators and denominators.

$= \frac{63}{28}$

Change to a mixed number.

$= 2\frac{7}{28}$

Simplify the fraction.

$= 2\frac{1}{4}$

Think Back

The reciprocal of a fraction is the fraction with its numerator and denominator flipped!

We find that the grapes will fill two big barrels and one quarter of another big barrel. You will need three big barrels!

Just as with multiplication, we first convert to improper fractions in order to divide mixed numbers.

Algorithm

To divide mixed numbers:

1. Convert everything to improper fractions.
2. Change the division sign to a multiplication sign. Take the reciprocal of the divisor.
3. Multiply the numerators and denominators.
4. Convert back to a mixed number in simplest form.

Let's do one more example.

Example

Find the quotient of $6\frac{2}{3} \div 4\frac{1}{4}$

Solution

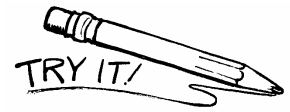
$$6\frac{2}{3} \div 4\frac{1}{4}$$

$$= \frac{20}{3} \div \frac{17}{4}$$

$$= \frac{20}{3} \times \frac{4}{17}$$

$$= \frac{80}{51}$$

$$= 1\frac{29}{51}$$



6. Find the quotients in simplest mixed number form.

a) $1\frac{1}{2} \div 1\frac{1}{4}$

b) $2\frac{3}{4} \div 6\frac{4}{7}$

7. 40 pounds of apples make $3\frac{1}{2}$ gallons of cider. How many pounds of apples are needed to make 1 gallon of cider?

☑ Review

1. Find the definitions of the following terms, and highlight them in the lesson.
    a. mixed number
    b. proper fraction
    c. improper fraction

2. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 7

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 7, Set A and Set B.

Set A

1. Convert mixed numbers to improper fractions, or improper fractions to mixed numbers in simplest form.

a) $2\frac{1}{2}$ b) $1\frac{7}{8}$ c) $4\frac{14}{15}$ d) $\frac{78}{3}$

2. Find the sum, difference, product, or quotient and express your answer in simplest mixed number form.

a) $1\frac{1}{2}+1\frac{1}{3}$ b) $1\frac{4}{5}\times 4\frac{1}{2}$ c) $2\frac{1}{3}-1\frac{3}{4}$

d) $3\frac{1}{5}\div 2\frac{1}{3}$ e) $9\frac{7}{8}\times 1\frac{2}{5}$ f) $8\frac{3}{4}-7\frac{1}{4}$

Set B

1. Put in order from least to greatest

$$\frac{15}{4},\quad 3\frac{1}{4},\quad 3\frac{1}{2}$$

2. You have $3\frac{2}{3}$ barrels of grapes, and you want to give $\frac{1}{3}$ of a barrel of grapes to each of your twelve aunts and uncles. Do you have enough grapes? *Explain* your answer.

3. Which is easier, adding fractions, or multiplying fractions? *Explain* why.

1. a) $\frac{4}{3}$ b) $\frac{23}{8}$ c) $\frac{15}{4}$ d) $\frac{28}{5}$

2. a) $1\frac{2}{3}$ b) $2\frac{5}{8}$ c) $1\frac{1}{4}$ d) $2\frac{1}{5}$

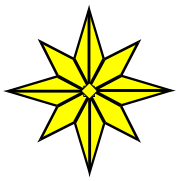
3. a) $3\frac{2}{3}$ b) $11\frac{3}{4}$ c) $8\frac{1}{15}$

4. a) $1\frac{2}{4} = 1\frac{1}{2}$ b) $\frac{8}{15}$ c) $1\frac{3}{5}$

5. a) $1\frac{2}{3}$ b) $18\frac{1}{14}$ c) $16\frac{1}{5}$

6. a) $1\frac{1}{5}$ b) $\frac{77}{184}$

7. $11\frac{3}{7}$



End of Lesson 7